Unit 2: Kinematics in 2D

4 - Projectile Motion Types 1 and 2

Remember that the x and y-components are perpendicular and therefore totally independent.

X-components

There is no Net Force working on the projectile in the X and the acceleration is always zero. Therefore the only equation we can ever use is: $\vec{V}_x = \frac{d_x}{t}$

Y-components

In this case there is always a constant acceleration of $-9.8\ m/s^2$. Because of this we need to use the Big Three!

$V_f = V_0 + at$

$d = V_0 t + \frac{1}{2}at^2$

$V_f^2 = V_0^2 + 2ad$

The only value that can ever be used on both sides is time because it is a scalar.

Time is the "gatekeeper" of projective problems

Problem Type 1:

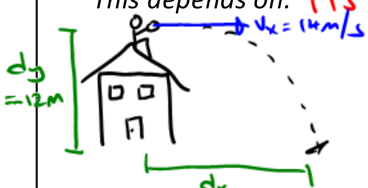
A student sits on the roof of their house which is 12 m high. She can launch water-balloons from a slingshot at 14.0 m/s. If she fires a water-balloon directly horizontally:

a. How long will it be airborne?

This depends on: it's height above the ground (d_y)

b. How far forward will it travel?

This depends on: it's horizontal velocity (V_x) and the time it's in the air (t)



x	y
$V_x = 14 m/s$	$V_{y0} = 0 m/s$
d_x?	$V_{yf} =$
$t = ?$ 1.565s	$d_y = -12m$
	$a_y = -9.8 m/s^2$
	$\boxed{t = 1.6s}$

$d_x = V_x \cdot t$

$d_x = (14 m/s)(1.565s)$

$\boxed{d_x = 22m}$

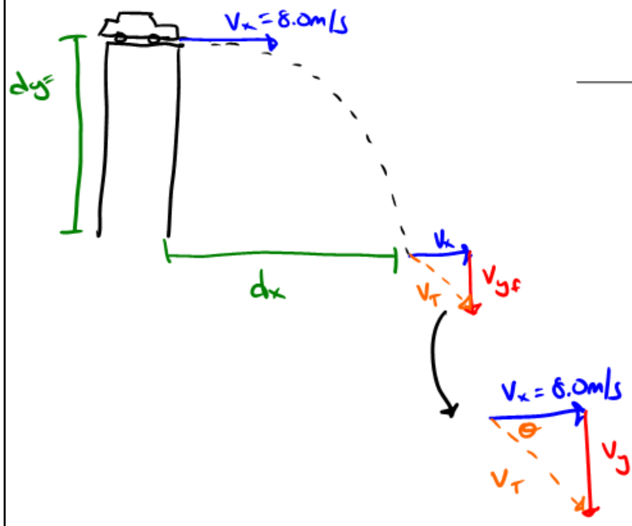
$d = \cancel{V_0 t} + \frac{1}{2}at^2$

$t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2(-12)}{-9.8}} = 1.565s$

Example: A Cutlass Supreme drives straight out of a parking garage at 8.0 m/s and hits the water 3.4 s later.

a. How far did the car fall?

b. What was his **total** impact velocity? (magnitude and direction)



X	Y
$V_x = 8.0 m/s$	$V_{yo} = 0 m/s$
$t = 3.4 s$	$V_{yf} = -33 m/s$
$d_x = ?$	$a_y = -9.8 m/s^2$
	$d_y = ? = -57 m$
	$t = 3.4 s$
	$d_y = \cancel{V_{yo} t}^{\,0} + \frac{1}{2}at^2$
	$d_y = \frac{1}{2}(-9.8)(3.4)^2 = -56.64 m$
	$V_{yf} = \cancel{V_{yo}}^{\,0} + at$
	$V_{yf} = at = (-9.8)(3.4) = -33.42 m/s$

$\tan\theta = \frac{V_y}{V_x}$

$\theta = \tan^{-1}\left(\frac{-33.42}{8}\right)$

$\theta = 76.50°$ → $\boxed{34 m/s @ 77° \text{ below the horizontal}}$

$V_T^2 = V_x^2 + V_y^2$

$V_T = \sqrt{(8.0)^2 + (-33.42)^2} = 34.27 m/s$

Problem Type 2: The Dukes of Hazzard are traveling at 85 km/h when they hit a jump that makes an angle of 25° above the horizontal.

a. How long are they airborne?

b. How far forward do they fly through the air?

c. What is their maximum height?

85 km/hr = 23.61 m/s

X	Y @ $T_{1/2}$
$d_x =$	$V_{yf} = 0 m/s$
$V_x = 21.40 m/s$	$V_{yo} = 9.978 m/s$
$t = 2.036 s$	$a_y = -9.8 m/s^2$
	$d_y =$
	$t_{1/2} = 1.018 s$
	$t = 2.036 s$

b) $d_x = v_x \cdot t$

$d_x = (21.40)(2.036)$

$d_x = 43.57 m$

$\boxed{d_x = 44 m}$

a) $V_{yf} = V_{yo} + at_{1/2}$

$t_{1/2} = \frac{V_{yf} - V_{yo}}{a} = \frac{0 - 9.978}{-9.8}$

$t_{1/2} = 1.018 s$; $\boxed{t = 2.036 s}$

c) $\cancel{V_{yf}}^{\,0 \text{ (at peak)}\,2} = V_{yo}^2 + 2ad_y$

$d_y = \frac{-V_{yo}^2}{2a} = \frac{-(9.978)^2}{2(-9.8)} = \boxed{5.1 m}$

$V_{yo} = 23.61 m/s (\sin(25°)) = 9.978 m/s$

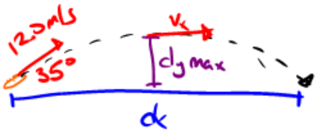
$V_x = 23.61 m/s (\cos(25)) = 21.40 m/s$

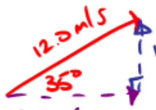
Example: A quarterback launches a ball to his wide receiver by throwing it at 12.0 m/s at 35° above horizontal.

a. How far downfield is the receiver?

b. How high does the ball go?

c. At what other angle could the quarterback have thrown the ball and reached the same displacement?





$V_{yo} = (12)\sin(35°) = 6.88\,m/s$

$V_x = (12)\cos(35°) = 9.83\,m/s$

X	Y @ $t_{1/2}$
$d_x =$	$V_{yo} = 6.88\,m/s$
$V_x = 9.83\,m/s$	$V_{yf} = 0\,m/s$
$t = 1.40\,s$	$a_y = -9.8\,m/s^2$
	$d_y =$

a) $d_x = v_x t$

$d_x = (9.83)(1.40)$

$d_x = 13.76\,m$

$\boxed{d_x = 14\,m}$

$t_{1/2} = 0.702\,s$; $t = 1.40\,s$

$\cancel{V_{yf}}^{0} = V_{yo} + a t_{1/2}$

$t_{1/2} = \frac{-V_{yo}}{a} = \frac{-6.88}{-9.8} = 0.702\,s$

b) $\cancel{V_{yf}}^{0\,2} = V_{yo}^2 + 2ad_y$

$d_y = \frac{-V_{yo}^2}{2a} = \frac{-(6.88)^2}{2(-9.8)} = \boxed{2.4\,m}$

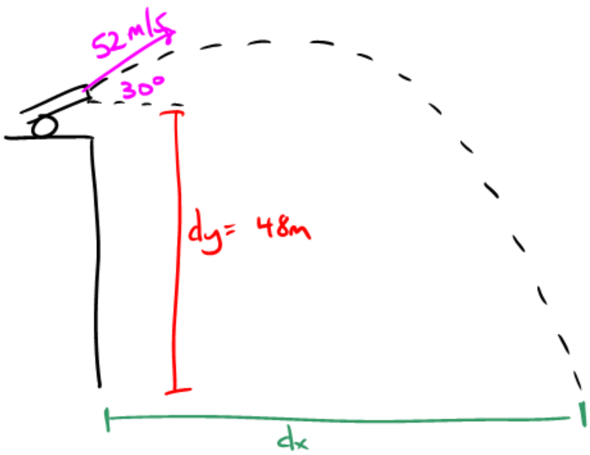
c) Complimentary Angles! $90° - 35° = \boxed{55°}$

Problem Type 3:

Ex: A cannon is perched on a 48 m high cliff. It aims 30° above the horizontal and fires a shell at 52 m/s. Find:

a) How long it takes for the sheel to hit the ground.

b) The distance it lands from the base of the cliff.



$52\,m/s$, $30°$

$V_{yo} = (52)\sin(30°) = 26.0\,m/s$

$V_x = (52)\cos(30°) = 45.0\,m/s$

x	y
$d_x =$	$V_{yo} = 26.0\,m/s$
$V_x = 45.0\,m/s$	$V_{yf} =$
$t = 6.76\,s$	$d_y = -48\,m$
	$a_y = -9.8\,m$
	$t =$

b) $d_x = V_x t = (45.0)(6.76)$

$\boxed{d_x = 304\,m}$

$V_{yf}^2 = V_{yo}^2 + 2ad$

$V_{yf} = \pm\sqrt{V_{yo}^2 + 2ad}$

$V_{yf} = \pm\sqrt{(26)^2 + 2(-9.8)(-48)}$

$V_{yf} = \pm 40.2\,m/s$

negative value! (going down)

a) $V_{yf} = V_{yo} + at$

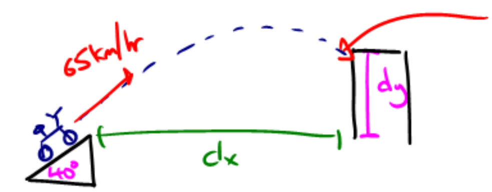
$t = \dfrac{V_{yf} - V_{yo}}{a} = \dfrac{(-40.2) - (26.0)}{-9.8}$

$\boxed{t = 6.76\,s}$

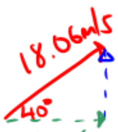
Ex: A BMXer leaves a ramp traveling at 65 km/h at a trajectory of 40° above the horizontal. ***After*** reaching his max height he strikes the top of a building 5.8 m above the ground.

a) What is the horizontal distance from the ramp to the building?

b) What is his speed when he hits the building?



x	y
$d_x = 38.6m$	$V_{yo} = 11.6 m/s$
$V_x = 13.8 m/s$	$V_{yf} =$
$t = 2.80s$	$a_y = -9.8 m/s^2$
	$d_y = -5.8m$
	$t = 2.80s$



$V_{yo} = (18.06)\sin(40°) = 11.6 m/s$

$V_x = (18.06)\cos(40°) = 13.8 m/s$

a) $d_x = v_x \cdot t = (13.8)(2.8)$

$\boxed{d_x = 38.6m}$

$V_{yf}^2 = V_{yo}^2 + 2ad$

$V_{yf} = \pm\sqrt{(11.6)^2 + 2(-9.8)(-5.8)}$

$V_{yf} = \pm 15.8 m/s$

choose (−)! going down

$V_{yf} = V_{yo} + at$

$t = \frac{V_{yf} - V_{yo}}{a} = \frac{-15.8 - 11.6}{-9.8}$

$t = 2.80s$

$V_T^2 = V_x^2 + V_{yf}^2$

$V_T = \sqrt{(13.8)^2 + (15.8)^2}$

$V_T = 21.0 m/s$

$\tan(\theta) = \frac{V_{yf}}{V_x}$

$\theta = \tan^{-1}\left(\frac{15.8}{13.8}\right) = 48.9°$

$\boxed{V_T = 21.0 m/s \text{ @ } 48.9° \text{ below the horizontal}}$