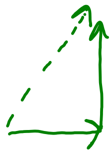


Vectors and Kinematics Notes

4 – Vector Addition and Subtraction

SCALAR	VECTOR
speed	velocity
time	displacement
distance	acceleration
mass	force
temperature	momentum
energy	

When we draw vectors we represent them as arrows.

Vector Addition

Whenever we add vectors we use...

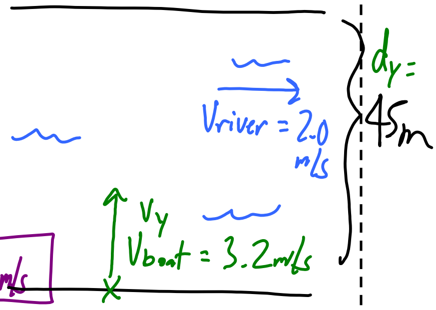
tip to tail method

To find the total or resultant vector, simply draw...

an arrow from the start to the finish

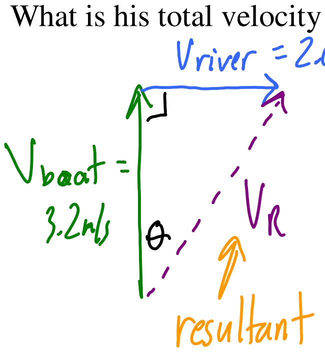
Ex: A student in a canoe is trying to cross a 45 m wide river that flows due East at 2.0 m/s. The student can paddle at 3.2 m/s

a. If he points due North and paddles how long will it take him to cross the river?



$V_y = \frac{d_y}{t}$ $t = \frac{d_y}{V_y} = \frac{45m}{3.2 m/s} = \boxed{14 s}$

b. What is his total velocity relative to his starting point in part a?



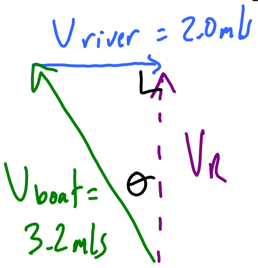
$V_R^2 = V_{boat}^2 + V_{river}^2$

$V_R = \sqrt{(3.2)^2 + (2.0)^2} = 3.77 m/s = \boxed{3.8 m/s}$

$\tan\theta = \frac{2.0}{3.2}$ $\theta = \tan^{-1}\left(\frac{2.0}{3.2}\right) = 32°$ E of N

$\boxed{3.8 m/s\ 32°\ E\ of\ N}$

c. If he needs to end up directly North across the river from his starting point, what heading should he take?



$\sin\theta = \frac{2.0}{3.2}$ $\sin^{-1}\left(\frac{2.0}{3.2}\right) = 39°$ W of N

d. How long will it take him to cross the river at this heading?

$V_{boat}^2 = V_R^2 + V_{river}^2$ $V_R = \sqrt{V_{boat}^2 - V_{river}^2} = \sqrt{(3.2)^2 - (2.0)^2} = 2.50 m/s$

$V_y = \frac{d_y}{t}$ $t = \frac{d_y}{V_y} = \frac{45 m}{2.50 m/s} = \boxed{18 s}$

<u>Vector Addition – Trig Method</u>

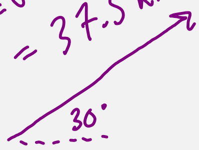
In the previous example we added perpendicular vectors which gave us a nice simple right triangle. In reality it's not always going to be that easy.

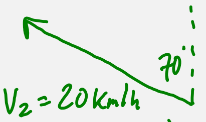
<u>Ex.</u> A zeppelin flies at 15 km/h 30° N of E for 2.5 hr and then changes heading and flies at 20 km/h 70° W of N for 1.5 hr. What was its final displacement?

$v_1 = 15\ km/h$

$d_1 = v \cdot t = (15)(2.5)$

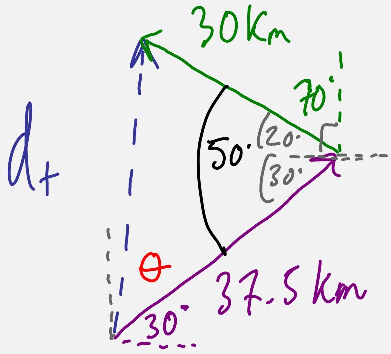
$= 37.5\ km$





$v_2 = 20\ km/h$

$d_2 = (20)(1.5) = 30\ km$



$c^2 = a^2 + b^2 - 2ab\cos C$

$= (37.5)^2 + (30)^2 - 2(37.5)(30)\cos 50°$

$\sqrt{c^2} = \sqrt{859.98}$

$c = 29.3\ Km$

$\frac{\sin\theta}{30} = \frac{\sin 50}{29.3}$

$\theta = \sin^{-1}\left(\frac{30 \sin 50}{29.3}\right)$

$= 52°$

$d_T = 29.3\ Km\ 82°\ N\ of\ E$

or

$8°\ E\ of\ N$

In order to solve non-right angle triangles, we will need to be familiar with the **Sine Law** and the **Cosine Law.**

Sine Law:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Cosine Law:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

<u>Vector Addition – The Component Method</u>

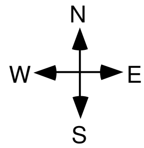
There is another method that we can use when adding vectors. This method is a very precise, stepwise approach, however it is the only way we can add 3 or more vectors.

- **Draw** each vector
- **Resolve** each vector into x and y components
- Find the **total sum** of x and y vectors
- **Add** the x and y vectors
- **Solve** using trig

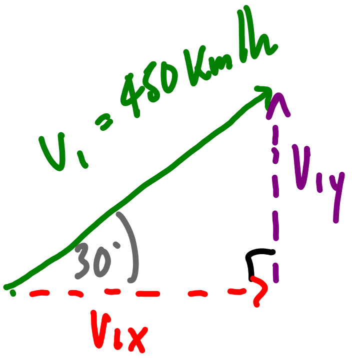
REMEMBER: When using x and y components…

- up and right are "+"
- down and left are "–"

Ex. An airplane heading at 450 km/h, 30° north of east encounters a 75 km/h wind blowing towards a direction 50° west of north. What is the resultant (total) velocity of the airplane relative to the ground?



Airplane vector:



x-component:

$\cos 30° = \frac{V_{1x}}{450}$

$V_{1x} = 450 \cos 30°$

$= 389.71$ km/h

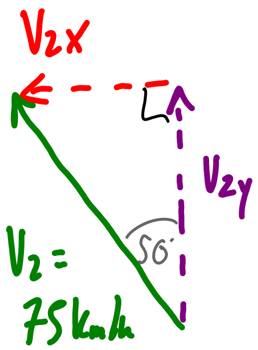
y-component:

$\sin 30° = \frac{V_{1y}}{450}$

$V_{1y} = 450 \sin 30°$

$= 225$ km/h

Wind vector:



x-component:

$\sin 50° = \frac{V_{2x}}{75}$

$V_{2x} = 75 \sin 50°$

$= -57.45$ km/h

to the left!

y-component:

$\cos 50° = \frac{V_{2y}}{75}$

$V_{2y} = 75 \cos 50°$

$= 48.21$ km/h

Adding the two vectors:

x-components of resultant:

$\sum V_x = V_{1x} + V_{2x}$

$= 389.71 + (-57.45)$

$= 332.26$ km/h

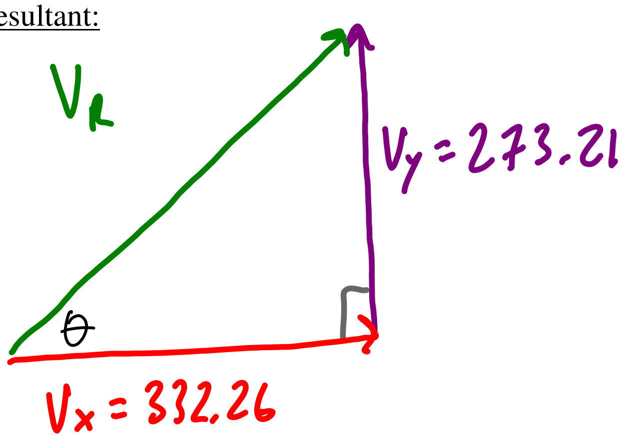
y-components of resultant:

$\sum V_y = V_{1y} + V_{2y}$

$= 225 + 48.21$

$= 273.21$ km/h

Total resultant:



$V_R = \sqrt{V_x^2 + V_y^2} = 430$ km/h

$\theta = \tan^{-1}\left(\frac{273.21}{332.26}\right) = 39°$ N of E

Vector Subtraction

With vectors a negative sign indicates that…it points in the exact opposite direction

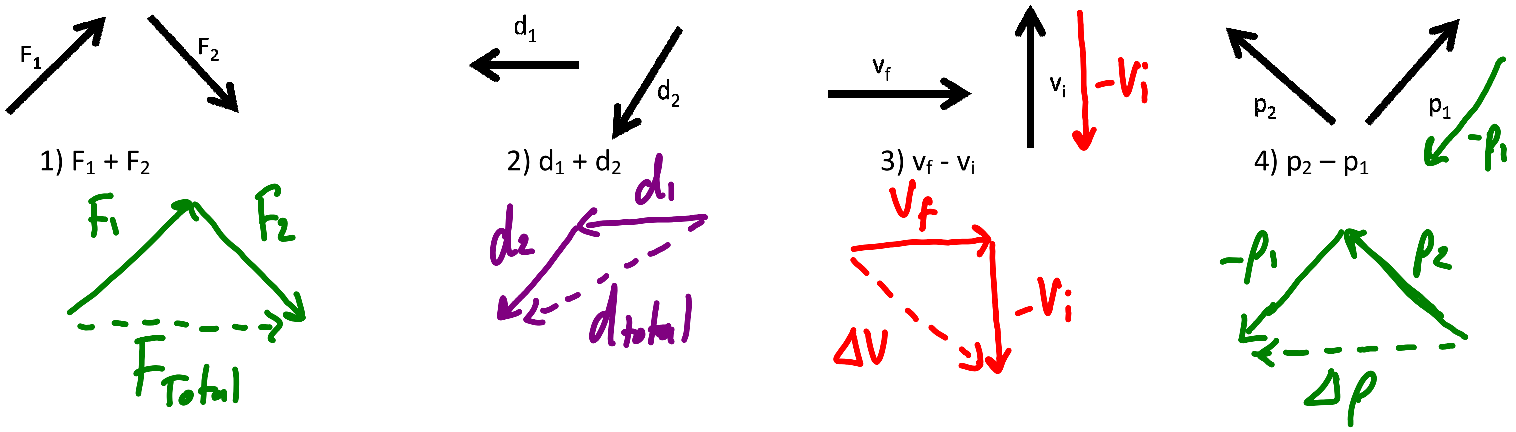
When subtracting vectors we still draw them *tip to tail*, except…we reverse the negative vector

We generally subtract vectors when dealing with a change in a vector quantity.

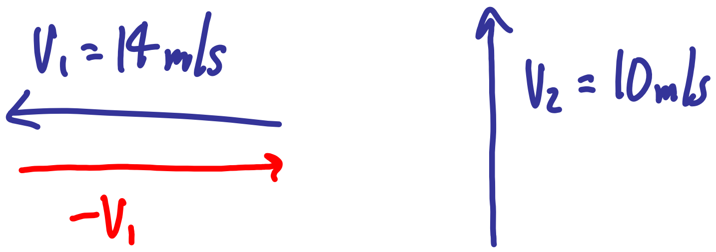
Recall:

Change = final − initial $\Delta\vec{V} = \vec{V}_f - \vec{V}_i$

Draw the Following



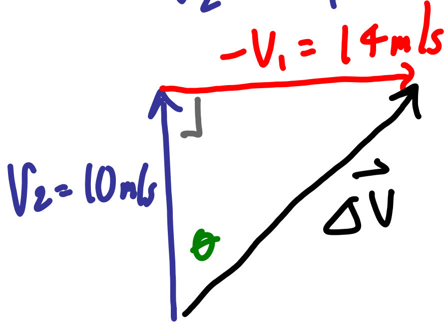
Ex: A cyclist is traveling at 14 m/s west when he turns due north and continues at 10 m/s. If it takes him 4.0 s to complete the turn what is the magnitude and direction of his acceleration?



$a = \frac{\Delta V}{t}$

$= \frac{V_f - V_i}{t}$

$\vec{\Delta V} = \vec{V_2} - \vec{V_1}$



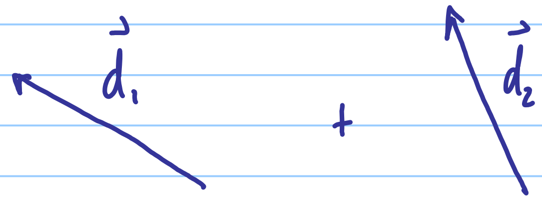
$\Delta V = \sqrt{V_2^2 + V_1^2} = 17.2\ m/s$

$\theta = \tan^{-1}\left(\frac{14}{10}\right) = 54^\circ$ E of N

$a = \frac{\Delta V}{t} = \frac{17.2\ m/s}{4.0\ s} = 4.3\ m/s^2$ 54° E of N

Summary of Methods

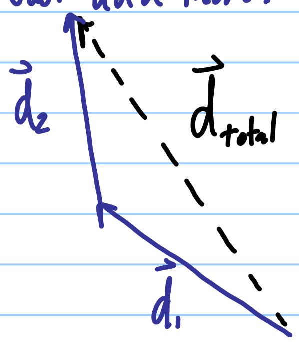
Say we want to add two vectors $\vec{d}_1 + \vec{d}_2$



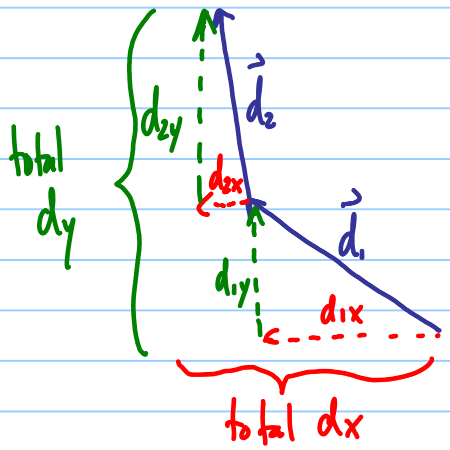
You have 2 choices

1) Trig Method

Just add them!



2) Component Method



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