AP Physics - One Dimensional Kinematics

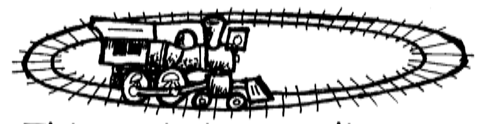
Velocity and speed are two closely related words. You might think that they are the same thing, but in physics we find that they are very different.

Speed is a measure of how fast something moves. It is a ***rate***. Rates are quantities divided by time. In addition, speed is a ***scalar*** quantity.

Velocity is also a rate – the rate that displacement changes with time. The really key thing here is that velocity is a ***vector***. It has magnitude – just as speed does – but it also has a direction. When we talk about speed, we don't care what about the direction of motion. The car went at a speed of 50 miles per hour. We don't care if it went south, north, east, west, whatever. With velocity we do care about the direction. Velocity would be the motion of a car that is going south at 35 mph.

A ***vector*** is a quantity that has both magnitude and direction.

A toy train traveling around a circular track is moving at a constant speed. It does not have a constant velocity, however, because its direction is constantly changing.



This train is traveling at a constant speed

Distance is a scalar – just how far you are from some point. ***Displacement***, on the other hand, is a vector – distance and direction.

Instantaneous velocity is the velocity of an object at any given instant of time. A car traveling from ***A*** to ***B*** does not always travel at a constant velocity - it stops, speeds up, slows down, etc. The speedometer on the dashboard reads out the instantaneous speed. At a stop sign it reads 0 mph, later on after the light turns green it might read 36 mph, and so on.

Average velocity is the velocity for an entire trip. It is the total distance divided by the total time.

$$\text{average velocity} = \frac{\text{total distane covered}}{\text{elapsed time}}$$

The symbol ***v*** is used for velocity (and is also used for speed). Some texts use $\bar{v}$, where a little bar is placed over the "v" indicating that it is a vector. We won't do that.

Average velocity is defined mathematically as: $$v = \frac{\Delta x}{\Delta t}$$

Δx means the change in ***x***, the displacement, Δt is the change in ***t***.

$$\Delta x = x - x_o$$

$$\Delta t = t - t_o$$

The subscript "***o***" means initial (it actually stands for "zero", representing the condition that you begin with). So Δx is the final displacement (or distance) minus the initial displacement. Other conventions can be used; $t_2 - t_1$, $t_f - t_i$, &tc.

If the initial conditions are zero, in other words, the motion started at time = 0 and at distance = 0, then the equation for average velocity can be shortened to:

$$v = \frac{x}{t}$$ This is also used when an object has a constant velocity.

We end up with three equations for average velocity, but they're all just variations of the same equation.

$$v = \frac{\Delta x}{\Delta t} \qquad v = \frac{x - x_o}{t - t_o} \qquad \text{or} \qquad v = \frac{x}{t}$$

It is very common to use other letters for displacement. For example, you might use ***s*** for some general displacement. You might use ***y*** if the motion is in the ***y*** direction. ***h*** is sometimes used if the distance is a vertical distance and ***r*** might be used if we're talking about the radius of a circle.

- In the 1988 Summer Olympics, Florence Griffith-Joyner won the 100 m race in a time of 10.54 s. Assuming the distance was laid out to the nearest centimeter so that it was actually 100.00 m, what was her average velocity in m/s and km/h?

Use the velocity equation: $v = \frac{x}{t}$, $\quad v = \frac{100.00\, m}{10.54\, s} = 9.48766 \frac{\text{m}}{\text{s}} = \boxed{9.488 \frac{m}{s}}$

Converting to km/h:

$$v = 9.488 \frac{\cancel{m}}{\cancel{s}} \left(\frac{1\, km}{1000\, \cancel{m}} \right) \left(\frac{3600\, \cancel{s}}{1\, h} \right) = \boxed{34.16 \frac{\text{km}}{h}}$$

- You begin a trip and record the odometer reading. It says 45 545.8 miles. You drive for 35 minutes. At the end of that time the odometer reads 45 569.8 miles. What was your average speed in miles per hour?

$$v = \frac{x - x_o}{\Delta t} = \frac{45\,569.8 \;\; mi - 45\,545.8 \;\; mi}{35 \text{ min}} = 0.6857 \frac{mi}{\text{min}}$$

$$v = 0.6857 \frac{mi}{\cancel{min}} \left(\frac{60\, \cancel{min}}{h} \right) = 41.14 \frac{mi}{h} = \boxed{41 \frac{mi}{h}}$$

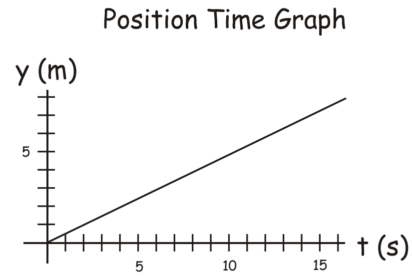
- A high speed train travels from Paris to Lyons at an average speed of 227 km/h. If the trip takes 2.00 h, how far is it between the two cities?

$$v = \frac{x}{t} \qquad x = vt \qquad x = 227 \frac{km}{\cancel{h}} \left(2.00\, \cancel{h} \right) = \boxed{454 \text{ km}}$$

All Motion Is Relative: Motion, i.e. velocity, is said to be relative. This is an important concept. What it means is that when we say that something has a given velocity, that velocity is relative to something else (these are called reference frames). So a car traveling to the east at 125 km/h is doing so relative to the earth. Sitting in the Kahuna Physics Institute, one is not moving - has no motion. This is true with respect to the room. However, the room and everything in it is rotating around the center of the earth. Not only that, but the earth itself is moving around the sun in its orbit! The solar system is moving around the center of the galaxy! The galaxy (and everything in it) is also moving away from the center of the universe!

If you are a passenger in an aircraft traveling at 500 mph over the earth, you are moving at 500 mph relative to the earth, but have no motion relative to the plane, unless you get up and go walking in the aisle, then you might have a motion, relative to the plane, of, say, 3 mph. Depending on which way you go, your motion relative to the earth could be 503 mph or 497 mph.

Position vs. Time Graphs: Let's look at a graph of position vs time:

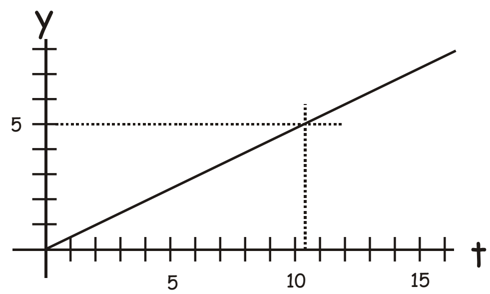


Displacement is plotted on the ***y*** axis and time is plotted on the ***x*** axis. The curve is a straight line. No doubt you recall that the equation for a straight line is:

$$y = mx + b$$

m is the slope and ***b*** is the ***y*** intercept

The slope is the change in ***y*** divided by the change in ***x***. (Otherwise known as "the rise over the run".)



Since we are graphing displacement on the ***y*** axis, the change in ***y*** is simply the change in displacement, or ***Δy***. We have ***Δt*** for the ***x*** axis. So the slope is:

$$m = \frac{\Delta y}{\Delta x} = \frac{\Delta y}{\Delta t}$$

But $\dfrac{\Delta y}{\Delta t}$ is the velocity ***v***!

Therefore the slope of the displacement Vs time graph is the velocity.

- What is the velocity of the object whose motion is depicted in this graph?

$$m = \frac{\Delta y}{\Delta x} = \frac{5\,m - 0}{10.4\,s - 0} = \boxed{0.48\,\frac{m}{s}}$$

Acceleration: When the velocity of an object is not constant, the rate at which it changes is defined as the acceleration. The symbol for acceleration is ***a***.

$$a = \frac{\Delta v}{\Delta t} \qquad a = \frac{v - v_o}{t - t_o} \qquad a = \frac{v}{t}$$

Acceleration is a vector quantity, just like velocity.

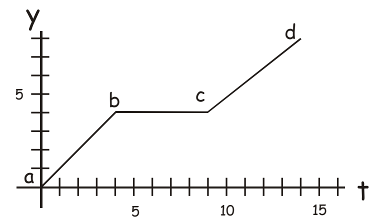
- A plane goes from rest to speed of 235 km/h in 15.0 s. Find the acceleration.

$$a = \frac{\Delta v}{\Delta t} = 235\,\frac{\cancel{km}}{\cancel{h}}\left(\frac{1}{15.0\,s}\right)\left(\frac{1\,000\,m}{1\,\cancel{km}}\right)\left(\frac{1\,\cancel{h}}{3\,600\,s}\right) = \boxed{4.35\,\frac{m}{s^2}}$$

4.35 m/s^2 means that the velocity changes by 4.35 m/s every second. At the end of the first second it is 4.35 m/s, after two seconds it is 8.70 m/s, after three seconds it is 13.0 m/s, after four seconds it would be 17.4 m/s, &tc.

- A car slows from 85.5 m/s to a speed of 33.2 m/s in 1.25 s. Find the acceleration.

$$a = \frac{\Delta v}{\Delta t} = \left(33.2\frac{m}{s} - 85.5\frac{m}{s}\right)\frac{1}{1.25\,s} = \boxed{-41.8\frac{m}{s^2}}$$

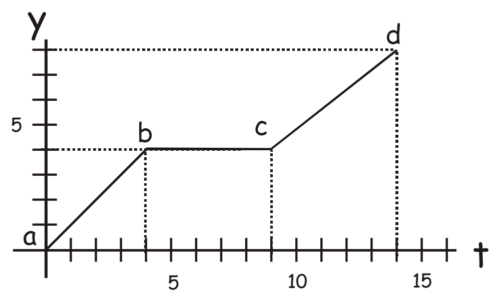


The minus sign means that the acceleration is in the opposite direction from the velocity.

We will always use m/s^2 for the units for acceleration. This is because things on earth don't accelerate for long periods of time. Very few things can actually accelerate for more than a few seconds.

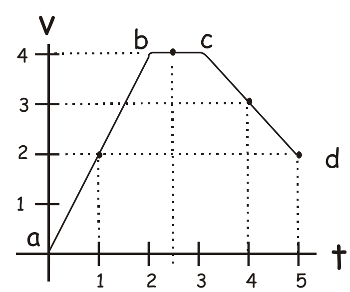
Let us now look at another position vs time graph.
In this graph, the slope of the graph changes from section ***ab*** to ***bc*** and then ***cd***. This means that the velocity has to change along these paths. The object moves at a constant velocity from zero displacement to a displacement of 4 m (this is from ***a – b***). This takes 4 seconds. Its velocity is a constant 1 m/s (the slope, right?). After the 4 seconds the object stops. It remains at rest for five seconds (***b – c***) and moves with a constant speed from time 9 s to time 14 s (***c – d***). Its velocity from 10 s to 15 s is:

$$v = \frac{\Delta y}{\Delta x} = \frac{8\,m - 4\,m}{14\,s - 9\,s} = 0.80\,\frac{m}{s}$$



Here's another example. This time we're looking at a velocity Vs time graph. During a baseball game a player runs after a fly ball. What is the player's acceleration from ***a*** to ***b***, ***b*** to ***c***, and ***c*** to ***d***?

The slope of the curve represents the acceleration (the Physics Kahuna asks you to convince yourself of this please).



a to ***b***: $a = \frac{v_f - v_i}{t_f - t_i}$ $\quad a = 2\frac{m}{s}\left(\frac{1}{1.0\,s}\right)$

$$a = \boxed{2.0\,\frac{m}{s^2}}$$

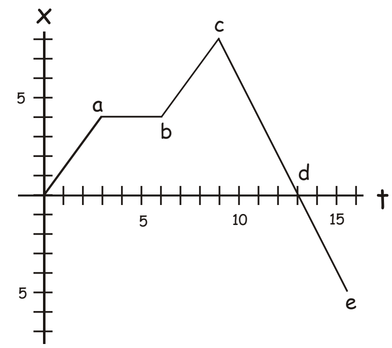
The player is initially at rest, the ball is hit and the player takes off to catch it. He accelerates from rest for four seconds. At the end of the four seconds, his velocity is 4 m/s.

b to ***c***: the slope is zero, so ***a*** is zero. The player is moving at a constant speed of 4 m/s for this portion of the graph.

c to **d**: $a = \frac{v - v_o}{t - t_o} = \left(2.0\frac{m}{s} - 4.0\frac{m}{s}\right)\left(\frac{1}{5.0\,s - 3.0\,s}\right) = \boxed{-\ 1.0\ \frac{m}{s^2}}$

The player is slowing down. He ends up moving at 2 m/s when he finally catches the ball.

- Here's another example; look at this graph:



Position Time Graph

Using this graph, find (a) the velocity from start to ***a***, (b) the velocity from ***a*** to ***b***, (c) the velocity from ***b*** to ***c***, (d) the velocity from ***c*** to ***d***, (e) the velocity from ***d*** to ***e***, (f) find the displacement after 7.0 s, (g) make a velocity Vs time graph for this system.

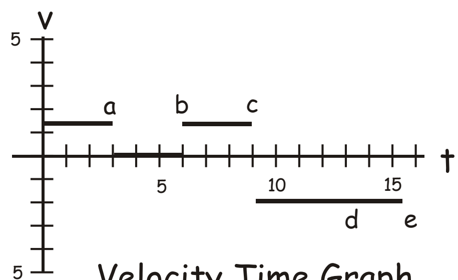
(a) 0 to ***a***: ***v*** = 1.3 m/s (b) ***a*** to ***b***: ***v*** = 0

(c) ***b*** to ***c***: ***v*** = 1.3 m/s (d) ***c*** to ***d***: ***v*** = - 2.0 m/s

(e) ***d*** to ***e***: v = - 2.0 m/s

(f) Displacement = 5.3 m

(g)



Velocity Time Graph

One Dimensional Motion, Constant Acceleration:

If a body is undergoing a constant acceleration, we can analyze the motion and come up with several equations that will describe the motion.

Start with the equation for acceleration:

$$a = \frac{v - v_0}{t - t_0} \quad \text{let} \quad t_o = 0, \quad v_i = 0 \qquad a = \frac{v - v_0}{t} \qquad v = v_0 + at$$

Here are two more important equations:

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0{}^2 + 2a\left(x - x_0\right)$$

So here are our three kinematic motion equations. These will be provided to you on the AP Physics Test. This is their form on the test equation sheet.

$$v = v_0 + at$$ ***v*** as function of time

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$ ***x*** as a function of time

$$v^2 = v_0^2 + 2a(x - x_0)$$ ***x*** as function of velocity

These equations simplify if initial conditions are zero. It is perfectly acceptable to use the simplified equations when solving problems.

$$v = at$$

$$x = \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2ax \quad \text{or} \quad v^2 = 2ax$$

- A car undergoes an average acceleration of 3.55 m/s^2. If the car is accelerated for 8.50 seconds, how far has it traveled?

$$x = \frac{1}{2}at^2 \qquad x = \frac{1}{2}\left(3.55\frac{m}{\cancel{s}^2}\right)(8.50\ \cancel{s})^2 = \boxed{128\ m}$$

- A truck undergoes an average acceleration of 2.50 m/s^2 as it speeds up. If it starts from rest, and accelerates for a distance of 875 m, how much time did it take to cover the distance?

$$x = \frac{1}{2}at^2 \qquad t^2 = \frac{2x}{a}, \qquad t = \sqrt{\frac{2x}{a}}$$

$$t = \sqrt{\frac{2(875\ \cancel{m})}{2.50\frac{\cancel{m}}{s^2}}} \qquad t = \sqrt{700.0\ s^2} = \boxed{26.5\ s}$$

- A car accelerates at 7.55 m/s^2. If it accelerates for 4.25 s, what speed does it reach?

$$v = v_o + at \qquad v = \left(7.55\frac{m}{s^2}\right)(4.25\ s) = \boxed{32.1\frac{m}{s}}$$

- A car accelerates at 9.00 m/s^2. If its initial velocity was 35.5 m/s and its final velocity is 107 m/s, what distance does it cover during the acceleration?

$$v^2 = v_o{}^2 + 2a(x - x_o) \quad = v_o{}^2 + 2ax \qquad\qquad x = \frac{1}{2a}\left(v^2 - v_o{}^2\right)$$

$$x = \frac{1}{2\left(9.00\,\frac{m}{s^2}\right)}\left(\left(107\,\frac{m}{s}\right)^2 - \left(35.5\,\frac{m}{s}\right)^2\right) \quad = \quad \boxed{566\,m}$$

Dear Doctor Science,
You're driving down the highway behind a vehicle that's doing only 50 mph. When you try to pass this vehicle, it speeds up to keep pace with you. Even if you're now going 100 mph, you still can't seem to pass this car. What's going on?
-- Karin S from Woodland Park, CO

Dr. Science responds:
Every slow moving car emits a force field that extends from the rear bumper of the car back to the car directly behind it. When you accelerate and approach the slow car, you compress the field. DeSoto's Third Law demands that a decrease in bumper force field size be compensated by an increase in axle rotation. So stop blaming the other guy; it's you who's in the driver's seat.

Kinematics in 1D

The Big 3 Kinematics Equations

If an object is accelerating then the formula:

$\vec{v} = \frac{\Delta \vec{d}}{\Delta t}$ Gives us only the avg. velocity

We can also find average velocity using:

v-bar (average) → $\bar{v} = \frac{\vec{v} + \vec{v}_o}{t}$

In order to solve problems with uniform acceleration we need to use 3 formulae. These 3 formulae use the variables:

v = final velocity
v_o = initial velocity
a = acceleration
d = displacement
t = time

1) $\vec{v} = \vec{v}_o + at$

Ex: A squad car traveling at 7.0 m/s East speeds up to 22.0 m/s East in 1.7 s. What is its acceleration?

$\vec{a} = \frac{\vec{v} - \vec{v}_o}{t}$

$\vec{a} = \frac{(22.0 m/s) - (7.0 m/s)}{1.7 s}$

$\vec{a} = 8.8 m/s^2 [E]$

2) $d = \vec{v}_o t + \frac{1}{2}at^2$

Ex: A sprinter starts from rest and accelerates uniformly. He travels 100.0 m south in 9.69 s, what was his average acceleration?

$d = v_o t + \frac{1}{2}at^2$ ($v_o = 0$)

$\vec{a} = \frac{2d}{t^2} = \frac{2(100.0 m)}{(9.69 s)^2}$

$\vec{a} = 2.13 m/s^2 [S]$

3) $v^2 = v_o^2 + 2ad$

Ex: A banana boat accelerates from 15.0 km/h at 2.00 m/s^2. How far has it traveled when it reaches 30.0 km/h?

15.0 km/h → 4.167 m/s; 30.0 km/h → 8.333 m/s

$v^2 = v_o^2 + 2ad$

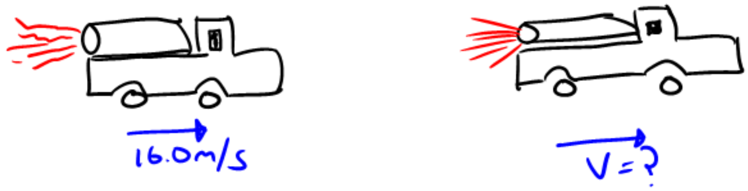
$2ad = v^2 - v_o^2$

$d = \frac{v^2 - v_o^2}{2a}$

$d = \frac{(8.333 m/s)^2 - (4.167 m/s)^2}{2(2.00 m/s^2)} = 13.0 m$

Ex 1: The Rocket Truck is traveling at 16.0 m/s when it is passed by a plane. It immediately hits the jets at accelerates at 14.0 m/s^2 for 3.25 s.

a. What final velocity does it reach?



$$\vec{v} = \vec{v}_0 + at$$

$$\vec{v} = (16.0\,m/s) + (14.0)(3.25) = \boxed{61.5\,m/s}$$

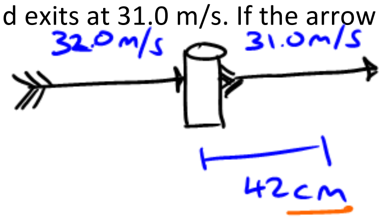
b. How far does it travel in this time?

$$d = v_0 t + \tfrac{1}{2}at^2$$

$$d = (16.0\,m/s)(3.25\,s) + \tfrac{1}{2}(14.0\,m/s^2)(3.25\,s)$$

$$\boxed{d = 126\,m}$$

Ex 2: An arrow strikes a can at 32.0 m/s and exits at 31.0 m/s. If the arrow is 42 cm long find its acceleration as it pierced the can. Ignore the width of the can.



$$v^2 = v_0^2 + 2ad$$

$$a = \frac{v^2 - v_0^2}{2d}$$

$$a = \frac{(31.0\,m/s)^2 - (32.0\,m/s)^2}{2(0.42\,m)}$$

$$\boxed{a = -75\,m/s^2}$$

Ex 3: A BMW and an F1 car both cross the finish line traveling at 200.0 km/h (→ 55.6 m/s). The BMW comes to a stop in 4.05 s and the F1 in 2.12 s. How much further did the BMW travel while stopping than the F1 car?

BMW

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$a = \frac{-\vec{v}_0}{t} = \frac{-55.6\,m/s}{4.05\,s} = -13.7\,m/s^2$$

$$d = v_0 t + \tfrac{1}{2}at^2$$

$$d = (55.6\,m/s)(4.05\,s) + \tfrac{1}{2}(-13.7\,m/s^2)(4.05\,s)^2$$

$$\boxed{d = 113\,m}$$

F1 car

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$a = \frac{-\vec{v}_0}{t} = \frac{-55.6\,m/s}{2.12} = -26.2\,m/s^2$$

$$d = v_0 t + \tfrac{1}{2}at^2$$

$$d = (55.6\,m/s)(2.12) + \tfrac{1}{2}(-26.2\,m/s^2)(2.12\,s)^2$$

$$\boxed{d = 59.0\,m}$$

Worksheet – The Big 3

A new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die and a new generation grows up that is familiar with it. – Max Planck

1. A racecar accelerates from rest to a speed of 287 km/h in 6.8 seconds. What is its average acceleration?

2. The space shuttle undergoes an acceleration of 53.9 m/s^2. How fast is it traveling at the end of 55.2 s?

3. You are in an elevator that is accelerating you upward at 4.55 m/s^2. How much time does it take you to reach a speed of 11.0 m/s?

4. A car is traveling at 108 km/h, stuck behind a slower car. Finally the road is clear and the car pulls over to make a pass. The driver stomps on the gas pedal and accelerates up to a speed of 135 km/h. If it took 3.5 s to reach this speed, what is the average acceleration of the car?

5. A driver has a reaction time of 0.50 s, and the maximum deceleration of her car is 6.0 m/s^2. She is driving at 20 m/s when suddenly she sees an obstacle in the road 50 m in front of her. Can she stop the car to avoid the collision?

6. Chameleons catch insects with their tongues, which they can rapidly extend to great lengths. In a typical strike, the chameleon's tongue accelerates at a remarkable 250 m/s^2 for 20 ms, then travels at a constant speed for another 30 ms. During this total time of 50 ms, how far does the tongue reach?

7. You're driving down the highway late one night at 20 m/s when a deer steps onto the road 35 m in front of you. Your reaction time before stepping on the brakes is 0.50 s, and the maximum deceleration of your car is 10 m/s^2. How much distance is between you and the deer when you come to a stop? What is the maximum speed you could have and still not hit the deer?

8. When a jet lands on an aircraft carrier, a hook on the tail of the plane grabs a wire that quickly brings the plane to a halt before it overshoots the deck. In a typical landing, a jet touching down at 240 km/h is stopped in a distance of 95 m. What is the magnitude of the jet's acceleration as it is brought to rest? How much time does the landing take?

9. A simple model for a person running the 100 m dash is to assume the sprinter runs with constant acceleration until reaching top speed, then maintains that speed through the finish line. If the sprinter reaches his top speed of 11.2 m/s in 2.14 s, what will be his total time?

Answers:

1. 12 m/s
2. 2980 m/s
3. 2.2 s
4. 2.14 m/s^2
5. Yes! She will come to a stop in 43.3m
6. 0.20 m or 20 cm
7. 5.0 m and 22 m/s
8. 23.4 m/s^2 and 2.85 s
9. 10.0s

AP Physics - Free Fall

Aristotle (384 – 322 BC), one of your basic ancient Greek philosophers, said that things fall because they want to regain their natural state - earth with earth, water with water, and so on. Thus a rock will fall back to the earth to be with the other rocks. Since a big rock possesses more "earth", it will fall faster than would, say, a feather (which is woefully inadequate in the earth amount category compared with your basic rock). Aristotle's idea appears to be true because a rock certainly falls faster than a feather. In fact it made so much sense, that Aristotle's ideas on the subject were the accepted truth for around 2 000 years until the Renaissance.

The first scientific study of gravity was done by Galileo Galilei (1564 - 1642). He was trained as a mathematician and was a university professor. In the late 1500's Galileo conducted a series of experiments on gravity. He is supposed to have demonstrated that heavy objects and light objects fall at the same speed. The act of doing experiments to find out what would happen – this was a very daring idea.

Here is Galileo's account of the experiment from his book, *Dialogues of two New Sciences*.

> "But I, Simplicio, who have made the test can assure you that a cannon ball weighing one or two hundred pounds or even more, will not reach the ground by as much as a span ahead of a musket ball weighing only half a pound, provided both are dropped from a height of 200 cubits...the larger outstrips the smaller by two finger-breadths, that is, when the larger has reached the ground, the other is short of it by two finger-breadths.

Galileo did not, as is popularly believed, state that the objects would hit the ground at the same time – he understood air resistance. He did understand that without air resistance, the objects ***would*** fall at exactly the same rate.

Galileo wrote about doing the experiment as if he had done it several times, but it is not clear where or when he did it. The story that he dropped cannon balls from the Leaning Tower of Pisa has only one source, his last pupil and biographer, Vincenzo Vivani. He describes a very public event -- the entire university in attendance to witness the thing. But no one at the university ever mentioned witnessing the event. So whether Galileo did or did not do the experiment is sort of up in the air.

Galileo's idea that things fall at the same rate flies in the face of common sense. It seems reasonable that heavy things ought to fall faster than light ones

To study gravity, Galileo found that he had to slow it down. This was because he couldn't measure the time it took an object to fall with the crude instruments of the time. Gravity was "slowed down" by having balls roll down inclined planes (ramps). Gravity still caused the motion, but its effect was decreased to the point where Galileo could gather useful data. Galileo found that the distance that accelerated objects would travel was proportional to the square of the time. More on this later.

Acceleration of Gravity: On the earth, gravity exerts a force on everything with mass. (A ***force*** is a push or pull.) The force makes all objects accelerate downwards, towards the center of the earth. This acceleration varies a tiny little bit depending on where you are - at the North Pole this acceleration is 9.83217 m/s^2 and at the Equator it has a value of 9.78039 m/s^2. This is because the earth is not a perfect sphere. Fortunately we can safely ignore the tiny differences in the acceleration of gravity. The value which is commonly used for this acceleration is 9.80 m/s^2. In English units it is 32.0 ft/s^2. Gravity's acceleration is kind of special so it is given its very own little symbol, ***g***.

$$g = \quad 9.80\,\frac{m}{s^2}$$

Drop a rock from the top of a cliff and, in one second, it will reach a speed of 9.80 m/s, after two seconds it will be traveling at 19.6 m/s, in three seconds it's going 29.4 m/s, at four seconds it's speed will be up to 39.2 m/s, and so on. It looks like the rock will keep going faster and faster and faster until it smashes into the earth, and it would, if it were falling in a vacuum. The thing is, see, that the air causes a frictional force that opposes the rock's fall and slows it down. For short drops with dense objects (like rocks) we can reasonably ignore the effects of the air. Oh, the fancy, scientific term for this force exerted by the air is ***drag*** or ***air resistance***, sometimes it is called ***wind resistance***. At high velocities or over long distances, the drag can become significant, especially for objects that are not dense, like feathers or leaves or fluff. In the real world, an object in free fall will accelerate to its ***terminal velocity***. This is the speed at which the force of gravity equals the drag force. The object then stops accelerating and falls at a constant velocity. People jumping out of airplanes experience this. The typical laid out position that sky divers use gives them a terminal velocity of around 100 mph.

When an object is released and allowed to fall, its motion can be described by the following table (ignoring air resistance):

Time	Velocity	Distance
1 s	9.8 m/s	4.6 m
2 s	19.6 m/s	19.6 m
3 s	29.4 m/s	44.1 m
4s	39.2 m/s	78.4 m
5s	49.0 m/s	122 m
&tc		

The kinematic acceleration equations can be used to describe the motion of falling objects.

- A ball is thrown straight upward. If it takes 4.25 seconds to reach the top of its path, what is its initial speed?

Since the ball is traveling upward, and the acceleration is downward, the ball will slow down as it moves up. For the upward part of its motion, its final velocity will be zero – it will then momentarily come to rest and then change direction and begin to accelerate downward. Since we know that for the upward part of its journey the final velocity is zero, we can easily calculate the initial velocity.

$$\cancel{v} = v_0 + at \quad v_0 = -at \quad = -\left(-9.80\frac{m}{s^{\cancel{2}}}\right)(4.25\,\cancel{s}) \quad = \quad \boxed{41.6\frac{m}{s}}$$

The velocity and acceleration have opposite directions, so one of the quantities must be negative. We've chosen "down" as the negative direction for the above solution (but hey, you could choose up to be negative if you like).

- A stone is thrown straight up from top of building with an initial speed of 35.5 m/s. (a) How high does it go from the top of the building? (b) How much time to reach the maximum height? (c) If the building is 45.2 m tall, how much time will it take to hit the ground from when it was initially launched?

(a) $v^2 = v_o^2 + 2ay$ $\quad$ v at top is zero so; $\quad$ $0 = v_o^2 + 2ay$

$$2ay = -v_o^2 \qquad y = \frac{-v_o^2}{2a} \quad = -\left(35.5\frac{m}{\cancel{s}}\right)^2\left(\frac{1}{2\left(-9.80\frac{\cancel{m}}{\cancel{s}^2}\right)}\right) \quad = \quad \boxed{64.3\ m}$$

(b) $v = v_o + at$ $\qquad$ $0 = v_o + at$ $\qquad$ $t = \dfrac{-v_o}{a}$

$$t = -35.5\frac{m}{s}\left(\frac{1}{-9.80\frac{m}{s^2}}\right) = \boxed{3.62\ s}$$

(c) ***The stone takes 3.62 s to reach the highest point in its path, it then must fall 64.3 m (to the top of the building) and then another 45.2 m to hit the deck below. So figure the problem from the top of the ball's path, where its velocity is zero and just before it begins to fall back down.***

It's initial velocity is zero, and, since the stone will be falling down, we can, what the heck, assume that down is positive (we can do this! We are in charge!):

$$y = \frac{1}{2}at^2 \qquad t = \sqrt{\frac{2y}{a}} \qquad t = \sqrt{\frac{2(64.3\ m + 45.2\ m)}{9.80\frac{m}{s^2}}} = 4.73\ s$$

The total time for the ball to be in the air is $3.62\ s + 4.73\ s = \boxed{8.35\ s}$

Negative or Positive: You get to select the coordinate system that you use to solve problems. This means you get to decide where the displacement is zero and what direction will be positive or negative. Look at what happens if you have a negative acceleration, such as **– 9.8 *m/s²***. Does this mean the object is decelerating (slowing) or does it mean that the object is moving along a negative (perhaps the ***y***) axis? It would depend on the problem. For an object moving on the ***x***-axis it would mean decelerating. For an object falling along the ***y***-axis, due to gravity, it means the object is accelerating, but in the downward direction. You choses you your directions for this stuff basically so that the calculations and everything are easiest.

Dear Doctor Science,
Why does a Kleenex, when dropped over a waste basket, always end up on the floor instead of the bottom of the basket?
-- Kevin Gustafson, M.D. from Minneapolis, MN

Dr. Science responds:
Kleenex is a brand name facial tissue, enjoying relatively high status compared to generic brands and lowly toilet paper. So the haughty tissue considers life in a waste basket, even after use, repugnant. Thus, you have two options if you want to avoid picking up tissues from around the basket. Either purchase a lower grade of facial tissue, or get a nicer waste basket. I've seen some copper lined ones with a tooled leather exterior that would attract even the top of the tissue pecking order.

NAME:

Free Fall Mini-Lab

Predictions: Guess how the following factors affect falling objects:

Surface Area: Mass:

Part 1: Paper sheet vs. Paper ball

Drop the each one and observe.

1) Do they hit at the same time or different times? Explain why.

2) What can you conclude about the effect of **shape** on falling objects?

Part 2: Marble vs. Tennis Ball

Drop both at the same time and observe when they hit the ground.

3) Do they hit at the same time or different times? Explain why.

4) What can you conclude about the effect of **mass** on falling objects?

Part 3: Determine acceleration due to gravity

Drop the Tennis ball and record your results.

Tennis Ball:

d = ______________

t = ______________

Use the kinematics formulas to calculate the acceleration of falling objects due to gravity:

Measured Acceleration Due to Gravity:	**Theoretical Acceleration Due to Gravity:**	**% difference =** $\left\lvert \frac{\text{Measured - Theoretical}}{\text{Theoretical}} \right\rvert$

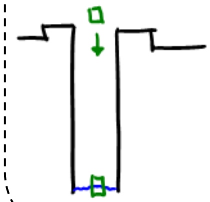
Kinematics in 1D

Acceleration Due to Gravity

- In the absence of air friction… all objects accelerate at the same rate
- Near Earth's surface the acceleration is

$-9.8\,m/s^2$

Example: A student drops their homework down a wishing well. After 2.4 s it hits the water at the bottom. How deep is the well?



$d = v_0 t + \frac{1}{2}at^2$ ($v_0 = 0$)

$d = \frac{1}{2}at^2$

$d = \frac{1}{2}(-9.8\,m/s^2)(2.4\,s)^2$

$d = -28\,m$

Example: A football is kicked straight up in the air at 15 m/s.

a) How high does it go?

$v^2 = v_0^2 + 2ad$

$d = \frac{v^2 - v_0^2}{2a} = \frac{(0)^2 - (15)^2}{2(-9.8)}$

$d = 11\,m$

b) What is its total hangtime?

$v = v_0 + at$

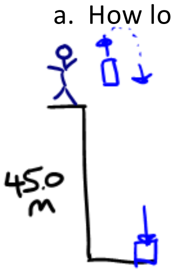
$t = \frac{v - v_0}{a}$

$t = \frac{-15\,m/s - 15\,m/s}{9.8\,m/s^2} = 3.1\,s$

Moral of the story on level ground…
- Time up = time down
- speed up = speed down

Example: A student stands on the edge of a 45.0 m high cliff. They throw their physics homework straight up in the air at 12.0 m/s.

a. How long does it take to come back down to the same height as the student?



$v = v_0 + at$

$t = \frac{v - v_0}{a} = \frac{(-12.0\,m/s) - (12.0\,m/s)}{-9.8\,m/s^2}$

$t = 2.45\,s$

b. If it falls all the way to the bottom of the cliff, how fast is it traveling when it hits the ground?

$v^2 = v_0^2 + 2ad$

$v = \sqrt{v_0^2 + 2ad}$

$v = \pm\sqrt{(12.0\,m/s)^2 + 2(-9.8\,m/s^2)(-45.0\,m)}$

$v = \pm 32.0\,m/s = -32.0\,m/s$

because we took the √

"+" doesn't make sense so only keep "−"

AP Physics – 3 - One Dimensional Kinematics with Free Fall HW– 3 Ans

1. A race car accelerates at a rate of 15.6 m/s². If it starts from rest, how much time till it traveling at 325 km/h?

$$a = \frac{v}{t} \qquad t = \frac{v}{a} \qquad t = 325\frac{km}{h}\left(\frac{1}{15.6\frac{m}{s^2}}\right)\left(\frac{1000\,m}{1\,km}\right)\left(\frac{1\,h}{3\,600\,s}\right) = \boxed{5.79\,s}$$

2. A truck falls off a cliff. If the cliff is 33.5 m high, how much time for the truck to reach the bottom?

$$x = \frac{1}{2}at^2 \qquad t = \sqrt{\frac{2x}{a}} \qquad t = \sqrt{2(33.5\,m)\left(\frac{1}{9.8\frac{m}{s^2}}\right)} = \boxed{2.61\,s}$$

3. You toss a ball straight up in the air, it goes up, comes down, and you catch it. If it took 5.6 s from when you threw it to when you caught it, how high did it go?

$$x = \frac{1}{2}at^2 \qquad x = \frac{1}{2}\left(9.8\frac{m}{s^2}\right)\left(\frac{5.6\,s}{2}\right)^2 = \boxed{38\,m}$$

4. The speed of sound is 344 m/s. You have built a really fantastic car that can really go fast. If the car can accelerate at 22.4 m/s, how much time till you reach the speed of sound? How many kilometers will you travel before you reach that speed?

$$a = \frac{v}{t} \qquad t = \frac{v}{a} \qquad t = 344\frac{m}{s}\left(\frac{1}{22.4\frac{m}{s^2}}\right) = \boxed{15.4\,s}$$

$$x = \frac{1}{2}at^2 \qquad x = \frac{1}{2}\left(22.4\frac{m}{s^2}\right)(15.4\,s)^2 = 2\,660\,m = \boxed{2.66\,km}$$

5. In 1947 Bob Feller, a pitcher for the Cleveland Indians, threw a baseball across the plate at 98.6 mph or 44.1 m/s. For many years this was the fastest pitch ever measured. If Bob had thrown the pitch straight up, how high would it have gone?

$$v^2 = v_0^2 + 2ax \qquad x = \frac{v^2}{2a} \qquad = \left(44.1\frac{m}{s}\right)^2 \frac{1}{2\left(9.8\frac{m}{s^2}\right)} = \boxed{99.2\,m}$$

6. You are on top of a building that is 75.0 m tall. You toss a ball straight up with an initial velocity of 33.8 m/s. How high does the ball travel? It goes up and then falls down to the ground below. How much time is it in the air?

Height: $$v^2 = v_o^2 + 2ax \qquad x = \frac{v^2}{2a} \qquad = \left(33.8\frac{m}{s}\right)^2\left(\frac{1}{2\left(9.8\frac{m}{s^2}\right)}\right) = \boxed{58.3\,m}$$

time up: $$v = at \qquad t = \frac{v}{a} \qquad = 33.8\frac{m}{s}\left(\frac{1}{9.8\frac{m}{s^2}}\right) = 3.45\,s$$

Time down:

$$x = \frac{1}{2}at^2 \qquad t = \sqrt{\frac{2x}{a}} \qquad t = \sqrt{2(58.3\,m + 75.0\,m)\left(\frac{1}{9.8\frac{m}{s^2}}\right)} = 5.22\,s$$

tot time: $$3.45\,s + 5.22\,s = \boxed{8.67\,s}$$

Vector and Kinematics Notes

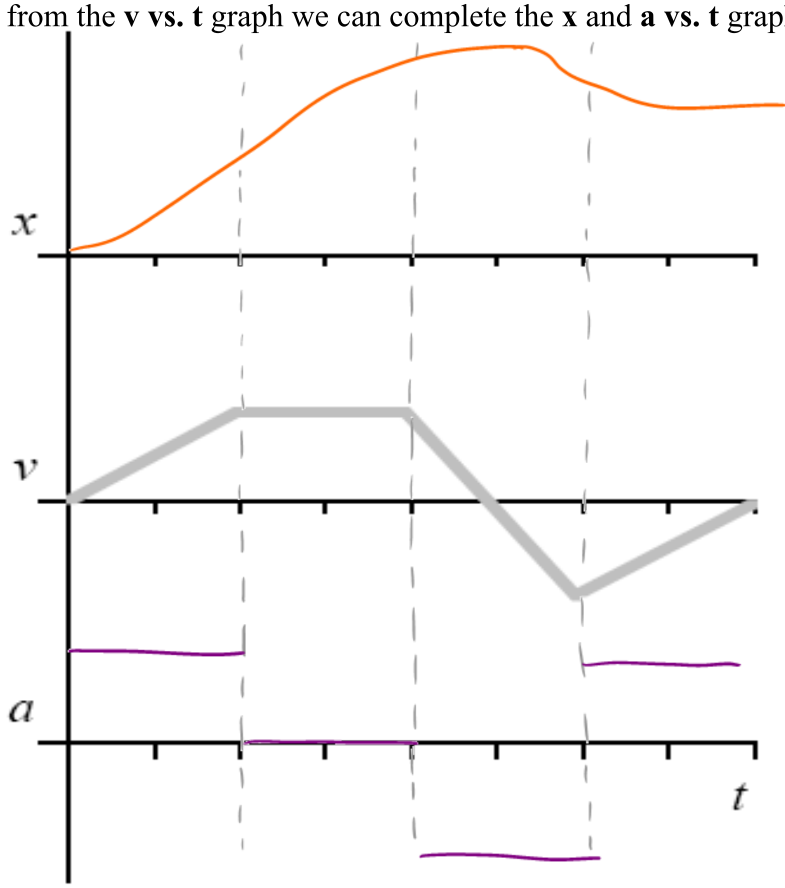
Graphs Revisited

There is certain information that can be taken from position vs. time (d vs. t) and velocity vs. time (v vs. t) graphs.

For Example:

d vs. t graphs:	v vs. t graphs:
slope = velocity	slope = acceleration area under the curve = displacement

Given the information from the **v vs. t** graph we can complete the **x** and **a vs. t** graphs

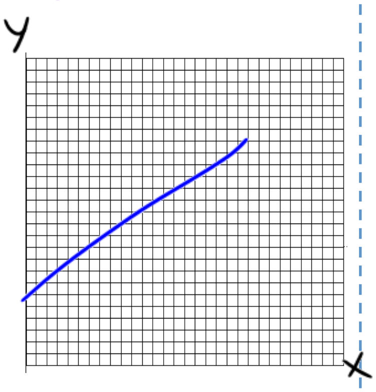


In Physics 12 you will be expected to perform more advanced graphical analysis on tests and in labs. EVERY time you make a graph you should follow the following rules.

- Label the axis
  - Independent variable on the x-axis
  - Dependent variable on the y-axis
- Give the graph an appropriate title. → Velocity vs. time for a cart moving down a ramp
- Scale each axis
  - Use… as much grid as possible
  - Choose a scale that is… easy to read
  -
- Plot the points and draw a line of best fit.
- Determine if the curve is linear or not

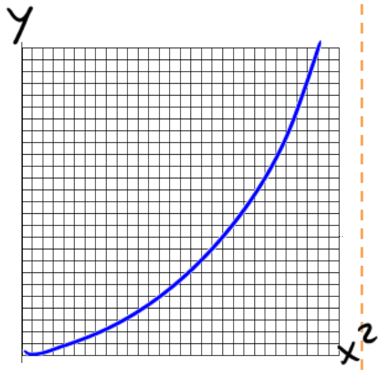
Direct:

$y \propto x$



Quadratic :

$y \propto x^2$

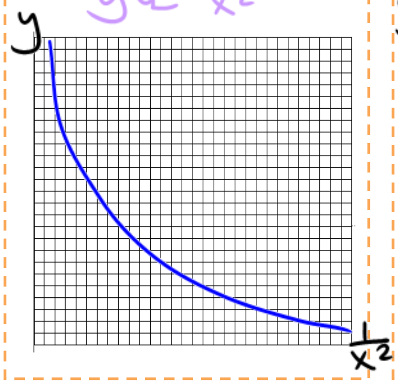


Inverse

$y \propto \frac{1}{x}$

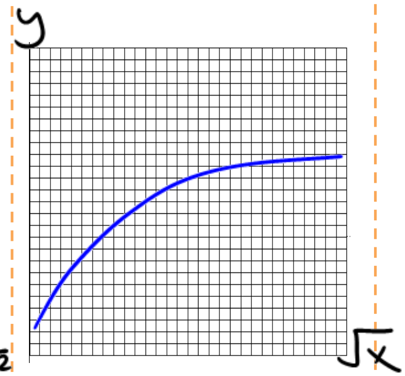
Inverse Square

$y \propto \frac{1}{x^2}$



Square Root:

$y \propto \sqrt{x}$



Finding Slope

To find the slope of a straight line:

- Choose… 2 points
- Choose them as… as far apart as possible
- Use only… points on the line NO DATA POINTS

Remember the equation of a line is:

$$y = mx + b$$

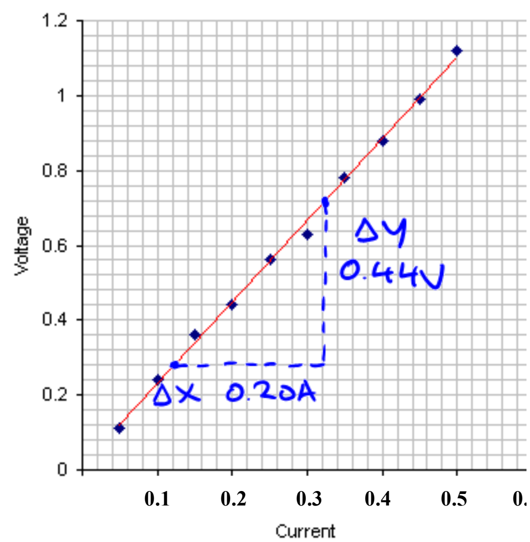
Determine the slope and y-intercept of the graph shown and write the equation describing this line.

$\text{slope} = \frac{\text{rise}}{\text{run}}$

$m = \frac{0.44\,V}{0.20\,A}$

$m = 2.2\,\Omega$

y-int = 0



Curve Straightening

Ex 1: A car starts at a certain speed and accelerates uniformly. A student collects data of velocity at different displacements.

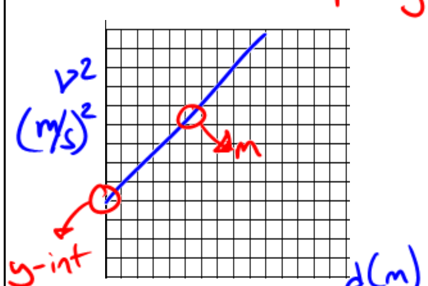
v	d
-	-
-	-
-	-
-	-

$v^2 = v_0^2 + 2ad$

So $v^2 \propto d$

$y = mx + b$

$v^2 = (2a)d + (v_0^2)$ — $2a$ → slope, v_0^2 → y-int



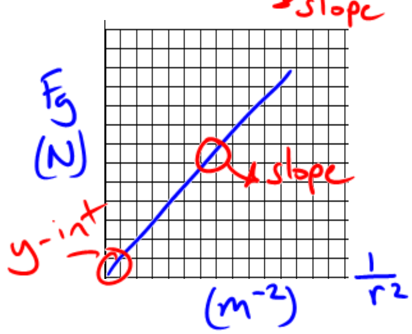
Ex 2: An astronaut standing on an asteroid measures the force of gravity acting on a 10 kg mass at different distances from the center of the asteroid.

Fg	r
-	-
-	-
-	-
-	-

$F_g = \frac{Gm_1m_2}{r^2}$ $\quad F_g \propto \frac{1}{r^2}$

$y = mx + b$

$F_g = (Gm_1m_2) \cdot \frac{1}{r^2}$ — Gm_1m_2 → slope



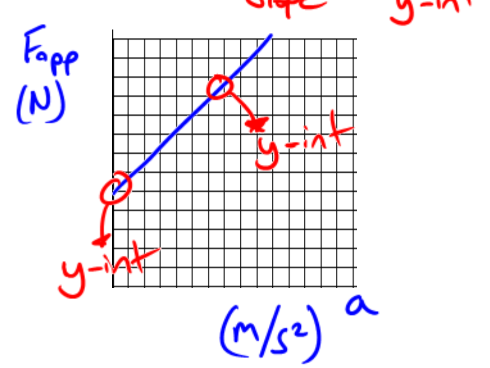
Ex 3: A student pushes a wooden block over a rough surface with different amounts of force and measures the acceleration each time.

Fapp	a
-	-
-	-
-	-
-	-

$F_{app} - F_f = ma$ $\quad F_{app} \propto a$

$y = mx + b$

$F_{app} = (m)a + (F_f)$ — m → slope, F_f → y-int



Graphing Worksheet

1) A student measures the acceleration of a lab cart as it moves at different speeds around a circular horizontal path. The data collected by the student is shown below:

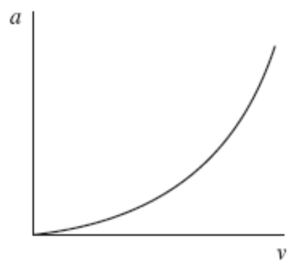
ACCELERATION (m/s^2)	5.7	12.9	25.2	40	49.7	72
VELOCITY (m/s)	2.0	3.0	4.2	5.3	5.9	7.1
Velocity^2 $(m/s)^2$	4.0	9.0	17	28	35	50

$v^2 = v_0^2 + 2ad$

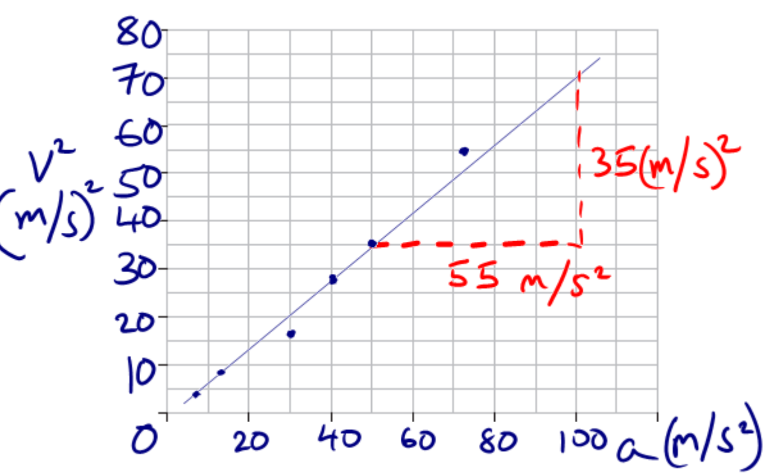
$v^2 = (2d)a + v_0^2$

$2d$ → slope; v_0^2 → y-int

When a graph of acceleration versus velocity is plotted a curve results as shown.



a) Manipulate the **velocity data** and use it to plot a straight line on the graph below.



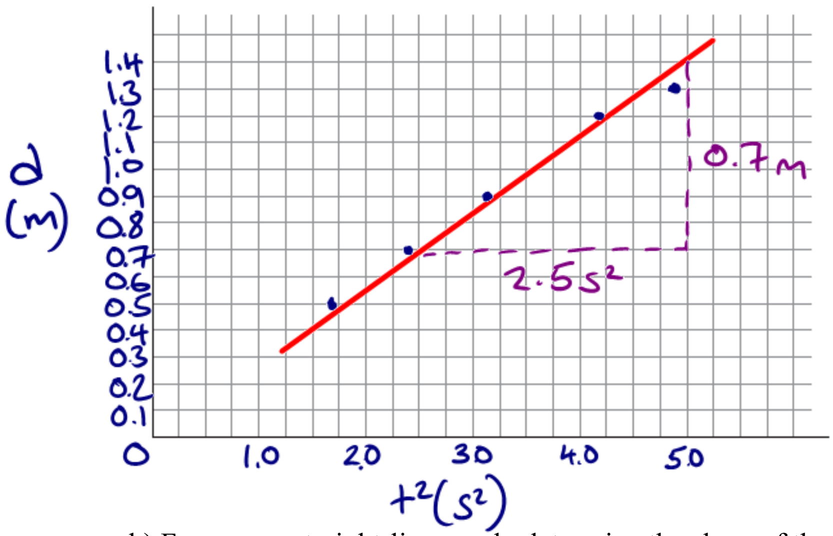
b) Calculate the slope of this graph including units.

$$slope = \frac{35\,(m/s)^2}{55\,m/s^2} = 0.64\,m$$

2) An experiment was performed on the surface of an asteroid. A mass was dropped from various heights and the time taken to fall was recorded.

d(m)	t(s)	t^2(s^2)
0	0	0
0.50	1.31	1.72
0.70	1.56	2.43
0.90	1.77	3.13
1.20	2.05	4.20
1.30	2.15	4.62

a) Plot a straight-line graph. ([redacted])



b) From your straight-line graph, determine the slope of the line. (Include units.)

$$\text{slope} = \frac{0.7\,m}{2.5\,s^2} = 0.28\,m/s^2$$

c) What is the acceleration due to gravity on the surface of this asteroid?

$d = v_0 t + \frac{1}{2}at^2$

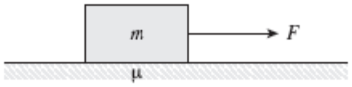
$d = \frac{1}{2}at^2$

$y = mx + b$

slope $= \frac{1}{2}a$ therefore $a = 2 \times$ slope

$a = 0.46\,m/s^2$

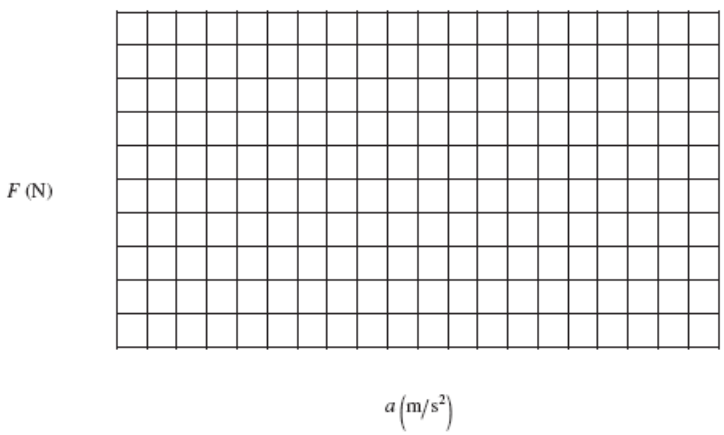
3) A force (F) was used to pull a wooden block across a floor as shown below.



The size of the force was varied and the data table below shows the size of the force and the block's resulting acceleration.

F (N)	a (m/s^2)
20	0.25
25	0.85
30	1.35
35	1.95

Plot the data on the graph below and draw a line of best fit. Extend the line back to the 'y' axis so that you have a y-intercept point and determine the slope of the line.

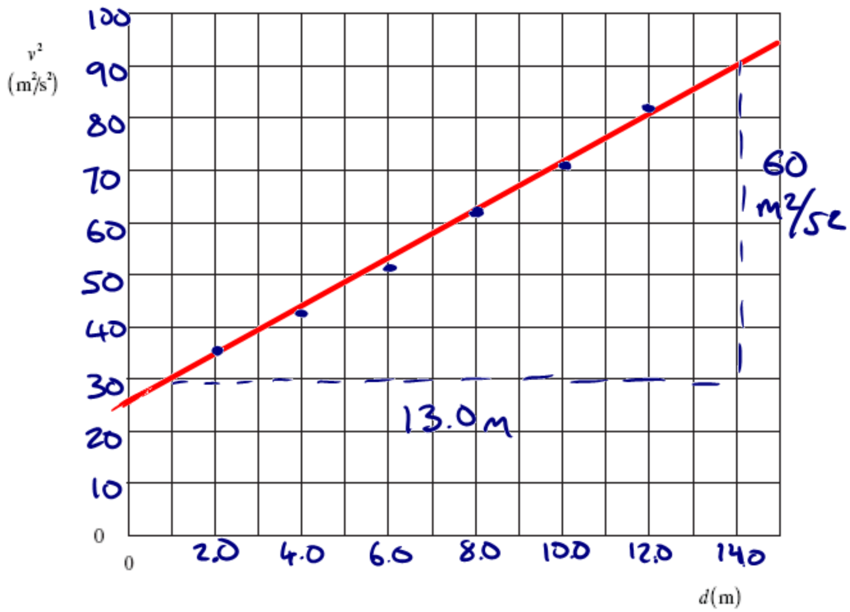


Using your slope value and your y-intercept value from the graph, determine the coefficient of friction between the block and the floor.

4) A student measures the final speed of an accelerating car at various displacements. The data collected is shown below.

FINAL SPEED (m/s)	v^2	DISPLACEMENT (m)
5.9	34.8	2.0
6.5	42.3	4.0
7.2	51.8	6.0
7.9	62.4	8.0
8.4	70.6	10.0
9.0	81.0	12.0

Plot a graph of the final speed squared, *v*2, versus the displacement, *d*, of the car on the graph below.



Determine the slope of the line of best fit to the data and state what the slope represents. Extend the line to the *y*-axis and use the *y*-intercept to determine the initial speed of the car.

$v^2 = v_0^2 + 2ad$

$v^2 = 2ad + v_0^2$

$y = mx + b$

$\text{slope} = \frac{60\ m^2/s^2}{13.0\ m} = 4.6\ m/s^2$

$a = \text{slope}/2 = \boxed{2.3\ m/s^2}$

$v_0 = \sqrt{y\text{-int}} = \sqrt{36.\ m^2/s^2} = \boxed{6.0\ m/s}$