## Unit 3: Kinematics in 2D

## 1 - Relative Velocity

In order to properly describe an object's motion we need to know...


Example: A man walks to the right with a velocity of $2 \mathrm{~m} / \mathrm{s}$ on a platform that moves with a velocity of $1 \mathrm{~m} / \mathrm{s}$ to the right.

a) What is the person's velocity relative to the platform? Reference frame: $\frac{p l a t f o r m}{2} \mathrm{~m} / \mathrm{s}$
$v_{\text {person-platform }}=1$
b) What is the person's velocity relative to the ground?


Example: You can throw a pie at $32 \mathrm{~m} / \mathrm{s}$. If you are standing on a train traveling $32 \mathrm{~m} / \mathrm{s}$ east and throw a pie forward what is its resultant (total) velocity?


Example: A bowling team on a train heads east at $15 \mathrm{~m} / \mathrm{s}$. A stationary observer watches them play as they pass. At what velocity would the following throws appear to be moving at?

Biff: Throws @ $12 \mathrm{~m} / \mathrm{s}$ East
Hank: Throws @ 18 mss East


Ralph: Throws @ $15 \mathrm{~m} / \mathrm{s}$ West

$$
\xrightarrow[V_{\text {train }}=15_{n} 1 s]{\stackrel{V_{\text {Ralph }}}{ }=15_{m}\left|s \quad V_{t_{0}} t_{a}\right|=0}
$$

Train A leaves Vancouver station traveling east at $90 . \mathrm{km} / \mathrm{h}$ at 9:00 am. At the same time train B leaves Montreal traveling west at $110 \mathrm{~km} / \mathrm{h}$. If the two stations are 4800 km .
a. At what time do they meet?
$V_{\text {rel }}=90 \mathrm{~km} / \mathrm{h}+110 \mathrm{kn} / \mathrm{h}=200 \mathrm{~km} / \mathrm{h}$
$V=\frac{d}{t} \quad t=\frac{d}{v}=\frac{4800 \mathrm{kn}_{n}}{200 \mathrm{k}_{\mathrm{n}} / \mathrm{h}}=24 \mathrm{~h}$
b. Where are they when they meet?

$$
\begin{aligned}
V_{A}=\frac{d_{A}}{t} \quad d_{A} & =V_{A} t=(90 \mathrm{~km} / \mathrm{h})(24 \mathrm{~h}) \\
& =2160 \mathrm{~km} E \text { of Vancouver }
\end{aligned}
$$

If the conductor of train A notices that is takes exactly 3.2 s for train B to pass it, what is the length of train $B$ ?

$$
\begin{aligned}
V_{v e} & =200 \mathrm{~km} / \mathrm{h} \div 3.6 \\
& =55.56 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

$$
d=V \cdot t
$$

$$
=(55.56 \mathrm{~m} / \mathrm{s})(3.2 \mathrm{~s})
$$

$$
=180 \mathrm{~m}
$$

WARNING: BEGINNER LEVEL AP QUESTION ALERT

A large cat, running at a constant velocity of $4.5 \mathrm{~m} / \mathrm{s}$ in the positive-x direction, runs past a small dog that is initially at rest. Just as the cat passes the dog, the dog begins accelerating at $0.5 \mathrm{~m} / \mathrm{s}^{2}$ in the positive- $x$ direction.
a. How much time passes before the dog catches up to the cat?
b. How far has the dog traveled at this point?
c. How fast is the dog traveling at this point?
a)

$$
\begin{aligned}
& d_{\text {cat }}=v_{\text {cat }} t \\
& d_{\text {dog }}=v_{p} t^{0}+\frac{1}{2} a_{\text {dog }} t^{2}
\end{aligned}
$$

$$
d_{c a t}=d_{\text {dog }}
$$

$$
v_{\text {cut }} \cdot t^{8}=\frac{1}{2} a_{\text {dg s }} t_{4.5}^{x}
$$



1. A plane is flying round trip to an destination 250 km North of its starting point. The plane flies with an airspeed of 325 $\mathrm{km} / \mathrm{h}$ and the wind is blowing at $50.0 \mathrm{~km} / \mathrm{h}$ due North.
a) How long does it take to get to the destination?
b) How long does it take to return to the starting point?
2) A tourist starts at the back of train that is 45 m long and walks towards the front at $1.5 \mathrm{~m} / \mathrm{s}$. The train is moving at 12 $\mathrm{m} / \mathrm{s}$.
a) How long does it take for the tourist to reach the front of the train, and how far has the tourist moved relative to the ground outside the train by the time they reach the front?
b) If the tourist decides to run all the way back to the end of the train at $6.0 \mathrm{~m} / \mathrm{s}$, how far have they travelled relative to the ground outside in this time?
3) Solve the following triangles (all sides and angles) using SOH - CAH - TOA and Pythagoras

4) Add the following $x$ and $y$ vectors, draw the resultant vector and solve its magnitude and direction.
a) $x: 3.4 \mathrm{~m} \quad \mathrm{y}: 2.7 \mathrm{~m}$
b) $x: 5.6 \mathrm{~m} / \mathrm{s} y:-7.1 \mathrm{~m} / \mathrm{s}$
c) $x:-211 m y:-44.0 m$

Unit 2: Kinematics in 2D Independence of Perpendicular Vectors


Example: After escaping from a maximum security stockade, the A-Team is trying to travel north across a 350 m river in a speed boat. The boat can travel at a speed of $25 \mathrm{~m} / \mathrm{s}$ in still water and the river flows to the east at $11 \mathrm{~m} / \mathrm{s}$.

Part 1: They point their boat directly north across the river.
a. What is their total (resultant) velocity?

b. How long does it take to cross the river?

$$
V=\frac{d}{t} \quad t=\frac{d_{y}}{V_{y}}=\frac{350_{m}}{25 \mathrm{~m} / \mathrm{s}}=14 \mathrm{~s}
$$



- Perpendicular vectors are...independent
- To find the total (resultant) vector we... do vector addition
- Don't forget that the resultant vector has... direction $\Rightarrow \theta$
- We don't use... + or for direction
Part 2: The Law has caught on to the boys and is waiting down river, on the other side.
a. At what heading should they point the boat so that they land safely, DIRECTLY across the river?
$V_{\text {boat }}=11 \mathrm{~m} / \mathrm{s}$

$$
\begin{aligned}
V_{R}=V_{y} & \theta=\sin ^{-1}\left(\frac{11}{25}\right) \\
& =26^{\circ} \text { Wot } N
\end{aligned}
$$

c. How far down-river do they end up?

$$
\begin{aligned}
V=\frac{d}{f} \quad d x=V_{x} t & =(11 \mathrm{~m} / \mathrm{s})(148) \\
& =154 \mathrm{~m} \\
& =150 \mathrm{~m} E
\end{aligned}
$$


b. How long will it take them to cross at this heading?

$$
=\frac{350 \mathrm{~m}}{22.45 \mathrm{~m} / \mathrm{c}}=16 \mathrm{~s}
$$

$$
\begin{aligned}
& V_{y}=\frac{d}{f} \\
& t=\frac{d x}{v_{y}} \\
& V_{\text {bot }}{ }^{2}=U_{R}^{2}+V_{\text {river }}{ }^{2} \\
& V_{k}=\sqrt{25^{2}-112^{2}}
\end{aligned}
$$

 4 - Vector Addition and Subtraction


When we draw vectors we represent them as
$\qquad$

Vector Addition
Whenever we add vectors we use...
tip to tail method
To find the total or resultant vector, simply draw...
an arrow from the start to the finish

Ex: A student in a canoe is trying to cross a 45 m wide river that flows due East at $2.0 \mathrm{~m} / \mathrm{s}$. The student can paddle at $3.2 \mathrm{~m} / \mathrm{s}$
a. If he points due North and paddles how long will it take him to cross the river?

$$
V_{y}=\frac{d_{y}}{t} \quad t=\frac{d_{y}}{V_{y}}=\frac{45 \mathrm{~m}}{3.2 \mathrm{~m} / \mathrm{s}}=14 \mathrm{~s}
$$

$\qquad$
to his starting point in part a?
b. What is his total velocity relative to his starting point in part a?

$\qquad$

## Vector Addition - Trig Method

In the previous example we added perpendicular vectors which gave us a nice simple right triangle.
In reality it's not always going to be that easy.

Ex. A zeppelin flies at $15 \mathrm{~km} / \mathrm{h} 30^{\circ} \mathrm{N}$ of E for 2.5 hr and then changes heading and flies at $20 \mathrm{~km} / \mathrm{h} 70^{\circ} \mathrm{W}$ of N for 1.5 hr . What was its final displacement?


In order to solve non-right angle triangles, we will need to be familiar with the Sine Law and the Cosine Law.

Sine Law:


$$
c^{2}=a^{2}+b^{2}-2 a b \cos C
$$

$$
=(37.5)^{2}+\left(30^{2}-2(37.5)(30) \text { coss } 5\right. \text { Cosine Law: }
$$

$$
c^{2}=a^{2}+b^{2}-2 a b \cos C
$$

$$
\frac{\sin \theta}{30}=\frac{\sin 50}{29.3}
$$

$$
\theta=\sin ^{-1}\left(\frac{30 \sin 5 t}{29.3}\right)
$$

$$
=52^{\circ}
$$

$$
\begin{array}{r}
d_{t}=29.3 \mathrm{~km} 82^{\circ} \mathrm{Nof} E \\
8^{\circ} \text { Eff } N
\end{array}
$$

## Vector Addition - The Component Method

There is another method that we can use when adding vectors. This method is a very precise, stepwise approach, however it is the only way we can add 3 or more vectors.

- Draw each vector
- Resolve each vector into x and y components
- Find the total sum of $x$ and $y$ vectors
- Add the $x$ and $y$ vectors
- Solve using trig

REMEMBER: When using $x$ and $y$ components...

- up and right are
- down and left are "-"

Ex. An airplane heading at $450 \mathrm{~km} / \mathrm{h}, 30^{\circ}$ nertheof east encounters a $75 \mathrm{~km} / \mathrm{h}$ wind blowing towards a direction $50^{\circ}$ west of north. What is the resultant velocity of the airplane relative to the ground?


y-component:

$$
\sin 30^{\circ}=\frac{V_{1 y}}{450}
$$

$$
V_{1 y}=450 \sin 30^{\circ}
$$

$$
\begin{aligned}
& =225 \mathrm{~km} / \mathrm{h}
\end{aligned}
$$



$$
\begin{aligned}
& \sin 50^{\circ}=\frac{V_{2 x} \text {-component }}{75} \\
& V_{2 x}=75 \sin 50^{\circ} \\
& =-57.45 \mathrm{~km} / \mathrm{h} \\
& \text { to the left! } \\
& \text { y-component: } \\
& \cos 50^{\circ}=\frac{V_{2 y}}{75} \\
& V_{2 y}=75 \cos 50^{\circ} \\
& =48.21 \mathrm{~km} / \mathrm{h}
\end{aligned}
$$

Adding the two vectors:
x-components of resultant:

$$
\begin{aligned}
\sum V_{x} & =V_{1 x}+V_{2 x} \\
& =389.71+(-57.45) \\
& =332.26 \mathrm{~km} / \mathrm{h}
\end{aligned}
$$

y-components of resultant:

$$
\begin{aligned}
\sum V_{y} & =V_{1 y}+V_{2 y} \\
& =225+48.21 \\
& =273.21 \mathrm{~km} / \mathrm{h}
\end{aligned}
$$

Total resultant:


$$
\begin{aligned}
V_{R} & =\sqrt{V_{x}^{2}+V_{y}^{2}}=430 K_{m} / h \\
\theta & =\tan ^{-1}\left(\frac{273.21}{332.26}\right)=390 \mathrm{~N}_{0} f E
\end{aligned}
$$

Vector Subtraction
With vectors a negative sign indicates that... it points in the exact opposite direction

When subtracting vectors we still draw them tip to tail, except... We reverse the negative vector We generally subtract vectors when dealing with a Change_ in a vector quantity.
Recall:

$$
\text { Change }=\text { final }- \text { initial } \quad \overrightarrow{\Delta V}=\overrightarrow{V_{f}}-\overrightarrow{V_{i}}
$$

Draw the Following


1) $F_{1}+F_{2}$


2) $d_{1}+d_{2}$


3) $v_{f}-v_{i}$


4) $p_{2}-p_{1}$


Ex: A cyclist is traveling at $14 \mathrm{~m} / \mathrm{s}$ west when he turns due north and continues at $10 \mathrm{~m} / \mathrm{s}$. If it takes him 4.0 s to complete the turn what is the magnitude and direction of his acceleration?


$$
\begin{aligned}
& \Delta V=\sqrt{V_{0}^{2}+V_{V}^{2}}=17.2 \mathrm{mls} \\
& \theta=\tan ^{-1}\left(\frac{14}{10}\right)=54^{\circ} \text { Got } \mathrm{N} \\
& a=\frac{\Delta V}{7}=\frac{17.2 \mathrm{nss}}{40 \mathrm{~s}}=4.3 \mathrm{~m} / \mathrm{s}^{2} 54 \text { Bot } \mathrm{N}
\end{aligned}
$$

Summary of Methods
Say we want to add two vectors $\vec{d}_{1}+\vec{d}_{2}$


You have 2 choices

1) Trig Mohod
2.) Component Mc hod

Jut add them!


Remember that the $x$ and $y$-components are perpendicular and therefore totally independent..

X-components
There is no $\qquad$ Net Force working on the projectile in the X and the acceleration is always
$\qquad$ . Therefore the only equation we can ever use is:

$$
\vec{V}_{+}=\frac{\vec{d}_{x}}{t}
$$

Y-components
In this case there is always a constant acceleration of
$\qquad$ need to use the $\qquad$ $V_{f}=V_{0}$ tat $d=v_{0} t+\frac{1}{2} a t^{2}$

$$
v_{f}^{2}=v_{0}^{2}+2 a d
$$

The only value that can ever be used on both sides is $\qquad$ time because it is a $\qquad$ Scalar time is the "gatekeeper" of projective problems

Problem Type 1:
A student sits on the roof of their house which is 12 m high. She can launch water-balloons from a slingshot at 14.0 $\mathrm{m} / \mathrm{s}$. If she fires a water-balloon directly horizontally:
a. How long will it be airborne?

This depends on: it's height above the ground (dy)
b. How far forward will it travel?

This depends on: it's horizontal velocity $\left(V_{x}\right)$ and the time it's in the cir $(t)$

$d x=(14 \mathrm{~m} / \mathrm{s})(1.5655)$

$$
d_{x}=22 \mathrm{~m}
$$

Example: A Cutlass Supreme drives straight out of a parking garage at $8.0 \mathrm{~m} / \mathrm{s}$ and hits the water 3.4 s later.
a. How far did the car fall?
b. What was his total impact velocity? (magnitude and direction)


Problem Type 2: The Dukes of Mazzard are traveling at $85 \mathrm{~km} / \mathrm{h}$ when they hit a jump that makes an angle of $25^{\circ}$ above the horizontal.
a. How long are they airborne?

| $x$ | $y @ T_{1 / 2}$ |
| :---: | :--- |
| $d_{x}=$ | $V_{y f}=0 \mathrm{~m} / \mathrm{s}$ |
| $V_{x}=21.40 \mathrm{~m} / \mathrm{s}$ | $V_{y y}=9.978 \mathrm{~m} / \mathrm{s}$ |

b. How far forward do they fly through the air?
$t=2.036 \mathrm{~s}$
c. What is their maximum height?
b) $d x=v_{x} \cdot t$
$d x=(21.40)(2.036)$
$\frac{d_{x}=43.57 \mathrm{~m}}{d_{x}=44 \mathrm{~m}}$

$V_{x}=23.61 \mathrm{~m} / \mathrm{s}(\cos (25))=21.40 \mathrm{~ms}$

$$
\begin{aligned}
& a_{y}=-9.8 \mathrm{~m} / \mathrm{s}^{2} \\
& d_{y}= \\
& t_{1 / 2}=1.018 \mathrm{~s} \\
& t^{2}=2.036 \mathrm{~s} \\
& a) v_{y f}=v_{y 0}+a t_{1 / 2} \\
& t_{1 / 2}=\frac{v_{y f}-v_{y-}}{a}=\frac{0-9.978}{-9.8} \\
& t_{1 / 2}=1.018_{s} j t=2.036 \mathrm{~s} \\
& \text { c) } v_{y f} q_{2} 0\left(a+p_{2}=v_{y 0}^{2}+2 a d_{y}\right. \\
& d_{y}=\frac{-v_{y 0}^{2}}{2 a}=\frac{-(9.978)^{2}}{2(-9.8)}=5.1 \mathrm{~m}
\end{aligned}
$$

Example: A quarterback launches a ball to his wide receiver by throwing it at $12.0 \mathrm{~m} / \mathrm{s}$ at $35^{\circ}$ above horizontal.
a. How far downfield is the receiver?
b. How high does the ball go?
c. At what other angle could the quarterback have thrown the ball and reached the same displacement?


$$
\text { a) } \begin{aligned}
d x & =v_{x}+ \\
d & =(9.83)(1.40) \\
d x & =13.76 \mathrm{~m} \\
d x & =14 \mathrm{~m}
\end{aligned}
$$

C) Complimentary Angles! $90^{\circ}-35^{\circ}=55^{\circ}$
b) $V_{y f} q^{2}=V_{y 0}^{2}+2 a d_{y}$

$$
d_{y}=\frac{-v_{y 0}{ }^{2}}{2 a}=\frac{-(6.88)^{2}}{2(-9.8)}=2.4 \mathrm{~m}
$$

Problem Type 3:
Ex: A cannon is perched on a 48 m high cliff. It aims $30^{\circ}$ above the horizontal and fires a shell at $52 \mathrm{~m} / \mathrm{s}$. Find:
a) How long it takes for the sheel to hit the ground.
b) The distance it lands from the base of the cliff.


$$
a_{y}=-9.8 \mathrm{~m}
$$

$$
\begin{aligned}
& v_{y f}{ }^{2}=v_{y}{ }^{2}+2 a d \\
& v_{y f}=+\sqrt{v_{y}{ }^{2}+2 a d} \\
& v_{y f}=+\sqrt{(26)^{2}+2(-9.8)(-48)}
\end{aligned}
$$

$$
\begin{aligned}
& 52 \mathrm{~m} / \mathrm{s} v_{y o}=(52) \sin \left(30^{\circ}\right)=26.0 \mathrm{~m} / \mathrm{s} \\
& 30^{\circ}-5(52) \cos \left(30^{\circ}\right)=45.0 \mathrm{~m} / \mathrm{s} \\
& v_{x}=(52
\end{aligned}
$$

$$
\text { b) } d_{x}=v_{x}+=(45.0)(6.76)
$$

$v_{y P}=26.0 \mathrm{~m} / \mathrm{s}$
$v_{y f}=$

$$
d_{y}=-48 \mathrm{~m}
$$

$$
t=
$$

$$
V_{y f}=\frac{ \pm 40.2 \mathrm{~m} / \mathrm{s}}{Q_{\text {negative }}}
$$

$$
\begin{aligned}
& \text { a) } \\
& v_{y f}=v_{y o}+a t \quad \text { negative volva! (yong) } \\
& t=\frac{v_{y f}-v_{y 0}}{a}=\frac{(40.2)-(26.0)}{-9.8} \\
& t=6.76 \mathrm{~s}
\end{aligned}
$$

Ex: A BMXer leaves a ramp traveling at $65 \mathrm{~km} / \mathrm{h}$ at a trajectory of $40^{\circ}$ above the horizontal. After reaching his max height he strikes the top of a building 5.8 m above the ground.
a) What is the horizontal distance from the ramp to the building?
b) What is his speed when he hits the building?


$$
V_{T}^{2}=V_{x}^{2}+V_{y f}^{2}
$$

$$
V_{T}=\sqrt{(13.8)^{2}+(15.8)^{2}}
$$

$$
V_{T}=21.0 \mathrm{~m} / \mathrm{s}
$$

$$
\tan (\theta)=\frac{v_{y f}}{v_{x}}
$$

$$
\theta=\tan ^{-1}\left(\frac{15.8}{13.8}\right)=48.9^{\circ}
$$

$$
V_{T}=21.0 \mathrm{~m} / \mathrm{s} @ 48.9^{\circ} \text { belowtre }
$$ horizontal

WARNING: AP LEVEL QUESTION ALERT
Evel Knievel is in a cannon with an initial height of 1.00 meter above the ground. When lit, the cannon fire's Evel at a speed of $26.6 \mathrm{~m} / \mathrm{s}$ at an angle of $50^{\circ}$ above the horizontal. Evel travels a horizontal distance of 60.0 meters to a 10.0 meter high castle wall. (Assume air friction is negligible.)
a. How long does it take Evil to reach a point directly about the wall?
b. Determine Evil's height (above the ground) at the time he travels over the wall.
c. Determine the velocity (magnitude and direction) of Evil as he passes over the castle wall.
d. If an exact clone of Evil is fired with the same initial speed at an angle of $55.5^{\circ}$ above the horizontal, will he clear the wall?
a) $t=\frac{d x}{V_{x}}=\frac{60.0 \mathrm{~m}}{17.1 \mathrm{~m} / \mathrm{s}}=3.51 \mathrm{~s}$ gate keeper. $11!\frac{x}{d x=60.0 \mathrm{~m}} \quad \underline{y}=-9.8 \mathrm{~m} / \mathrm{s}$

$$
\text { ( } \begin{aligned}
& V_{x}=26.6 \cdot \cos (50) V_{y_{y}}=26.6 \sin (50) \\
& V_{x}=17.1 \mathrm{~m} / \mathrm{s} \quad V_{y_{0}}=20.3 \mathrm{~m} / \mathrm{s} \\
& t=? ?
\end{aligned}
$$

$$
\begin{aligned}
& d_{y}=(1.0)+(20.3)(3.51)+(0.5)(-9.8)(3.51)^{2} \\
& d_{y}=11.9 \mathrm{~m}
\end{aligned}
$$

c) $\nabla_{x}$ is constant!

$$
\begin{aligned}
& V_{y f}=V_{y o}+g t=(20.3 \mathrm{~m} / \mathrm{s})+\left(-9.8 \mathrm{~m} / \mathrm{s}^{2}\right)(351 \mathrm{~s})=-14.1 \mathrm{~m} / \mathrm{s} \\
& \nabla_{x} \quad V_{r}=\sqrt{V_{x}^{2}+\nabla_{y f}^{2}} \quad \theta=\tan ^{-1}\left(\frac{V_{v_{f}}}{\nabla_{x}}\right) \\
& \nabla_{r} \quad \theta=39.5^{0} \\
& \nabla_{y f} \quad V_{r}=22.2 \mathrm{~m} / \mathrm{s} \quad \theta \text { horizontal }
\end{aligned}
$$

$22.2 \mathrm{~m} / \mathrm{s} @ 39.5^{\circ}$ below the horizontal
d) $\nabla_{x}=26.6 \mathrm{~m} / \mathrm{s} \cdot \cos (55.5)=15.1 \mathrm{~m} / \mathrm{s}$

$$
\begin{array}{ll}
\nabla_{x}=26.6 \mathrm{~m} / \mathrm{s} \cdot \cos (55.5)=15.1 \mathrm{~m} / \mathrm{s} & g=-9.8 \mathrm{~m} / \mathrm{s} \\
\nabla_{y 0}=21.9 \mathrm{~m} / \mathrm{s} \cdot \cos (55.5)=21.9 \mathrm{~m} / \mathrm{s}
\end{array}
$$

$$
g=-9.8 \mathrm{~m} / \mathrm{s}
$$

$$
t=\frac{d x}{\nabla_{x}}=\frac{60.0 \mathrm{~m}}{15.1 \mathrm{~m} / \mathrm{s}}=3.98 \mathrm{~s}
$$

$$
d_{y}=x_{0}+\nu_{y_{0}} t+\frac{1}{2} g t^{2}
$$

$d y=(1.0)+(21.9)(3.98)+(0.5)(-9.8)(3.98)^{2}$

