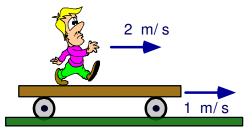
### Unit 3: Kinematics in 2D

1 - Relative Velocity

In order to properly describe an object's motion we need to know... its frame of reference, point of view

Example: A man walks to the right with a velocity of 2 m/s on a platform that moves with a velocity of 1 m/s to the right.



a) What is the person's velocity relative to the platform?

Reference frame: p|athorn $v_{person-platform} = 2 m/s$ 

b) What is the person's velocity relative to the ground?

Reference frame: \_\_\_\_\_\_  $v_{person-ground} = v_{platform} + v_{person-platform}$   $v_{person-ground} = 1 m/s + 2 m/s$   $v_{person-ground} = 3 m/s$ 

Example: You can throw a pie at 32 m/s. If you are standing on a train traveling 32 m/s east and throw a pie forward what is its resultant (total) velocity?

$$\frac{V_{\text{train}} - 32 \text{ m/s}}{V_{\text{train}} - 32 \text{ m/s}}$$

Example: A bowling team on a train heads **east** at 15 m/s. A stationary observer watches them play as they pass. At what velocity would the following throws appear to be moving at? Vtrain=15mls Ubik=12mls

Biff: Throws @ 12 m/s East

Hank: Throws @ 18 m/s East

Ralph: Throws @ 15 m/s West

Utram= ISm ls VRabb= LSm b Vtohl= 0

Vhank=18mls Vhank=18mls

Train A leaves Vancouver station traveling east at 90. km/h at 9:00 am. At the same time train B leaves Montreal traveling west at 110 km/h. If the two stations are 4800 km.

a. At what time do they meet?

$$V_{rel} = 90 k_m / h + 110 k_m / h = 200 k_m / h$$

$$V = \frac{d}{t} + \frac{d}{V} = \frac{4800 k_m}{200 k_m / h} = 29 h$$
b. Where are they when they meet?

$$V_{A} = \frac{d_{A}}{t}$$
  $d_{A} = V_{A}t = (90 \text{ km/h})(24 \text{ h})$   
= 2160 km E of Vancouver

If the conductor of train A notices that is takes exactly 3.2 s for train B to pass it, what is the length of train B? Vrel= 200Kn/h: 3.6 = 5556 1.

$$d = V + \frac{1}{(55.56 \text{ m/s})(3.2 \text{ c})} = \frac{180 \text{ m}}{3.2 \text{ c}}$$

### WARNING: BEGINNER LEVEL AP QUESTION ALERT

A large cat, running at a constant velocity of 4.5 m/s in the positive-x direction, runs past a small dog that is initially at rest. Just as the cat passes the dog, the dog begins accelerating at 0.5 m/s<sup>2</sup> in the positive-x direction.

- a. How much time passes before the dog catches up to the cat?
- b. How far has the dog traveled at this point?
- c. How fast is the dog traveling at this point?

a) deat = Veat. T ddog = vA + 12 adgt2 dcat = ddog Vcat . #= 2 adag +2 s.om/s 185 2 Vcat =  $t = \frac{2}{2}$ + tat b) ddog = V6  $d_{dog} = \frac{1}{2} (0.5 m/s) (20.5)^2$ KB/B M ddag C)  $V_{fcat} = V_{gcat} + a_{cat} + V_{fcat} = (0.5m/s^2)(200,s)$ VECat = XAP. m/s



### Worksheet – Independence of Perpendicular Vectors

1. A plane is flying round trip to an destination 250 km North of its starting point. The plane flies with an airspeed of 325 km/h and the wind is blowing at 50.0 km/h due North.

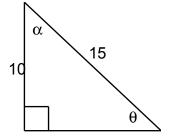
a) How long does it take to get to the destination? b) How long does it take to return to the starting point?

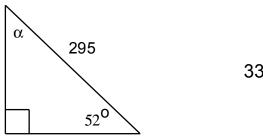
2) A tourist starts at the back of train that is 45 m long and walks towards the front at 1.5 m/s. The train is moving at 12 m/s.

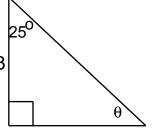
a) How long does it take for the tourist to reach the front of the train, and how far has the tourist moved relative to the ground outside the train by the time they reach the front?

b) If the tourist decides to run all the way back to the end of the train at 6.0 m/s, how far have they travelled relative to the ground outside in this time?

3) Solve the following triangles (all sides and angles) using SOH – CAH - TOA and Pythagoras







4) Add the following x and y vectors, draw the **resultant** vector and solve its **magnitude and direction**.

a) x: 3.4 m y: 2.7 m

b) x: 5.6 m/s y: -7.1 m/s

c) x: -211 m y: -44.0 m

# Unit 2: Kinematics in 2D Independence of Perpendicular Vectors

Example: After escaping from a maximum security stockade, the A-Team is trying to travel north across a 350 m river in a speed boat. The boat can travel at a speed of 25 m/s in still water and the river flows to the east at 11 m/s.

Part 1: They point their boat directly north across the river.

a. What is their total (resultant) velocity?  

$$V_{R}^{2} = V_{bool}^{2} + V_{river}^{2} \quad fan \theta = \frac{V_{Rev}}{V_{bool}}$$

$$V_{R} = \sqrt{2S^{2} + 11^{2}} \quad \theta = \frac{1}{V_{bool}} - \frac{1}{(\frac{11}{2S})}$$

$$= 27.31 \text{ m/s} = 29^{\circ}$$

$$= 27 \text{ m/s} 24^{\circ} \text{ E of } N$$
b. How long does it take to cross the river?  

$$V = \frac{1}{T} \quad T = \frac{1}{V_{Y}} = \frac{350 \text{ m}}{25 \text{ m/s}} = 145$$

$$V_{y} = V_{bon} t = 1$$

- Perpendicular vectors are...independent
- To find the total (resultant) vector we...do vector addition
- Don't forget that the resultant vector has... direction => +

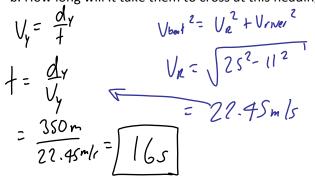
Univer = 11 m/s  $V_{\text{boot}} = 1$  25 m/s  $V_{\text{boot}} = 1$   $V_{\text{boot}} = 1$  $V_{\text{boot}}$ 

 $= \int SO_m E$ <u>Part 2</u>: The Law has caught on to the boys and is waiting down river, on the other side.

a. At what heading should they point the boat so that they land safely, DIRECTLY across the river?

V boot = 11 mls Sin 
$$\theta = \frac{11}{25}$$
  
V boot =  $\frac{1}{25}$   
V boot =  $\frac{1}{25}$   
V boot =  $\frac{1}{25}$   
 $\frac{1}{25}$   

b. How long will it take them to cross at this heading?



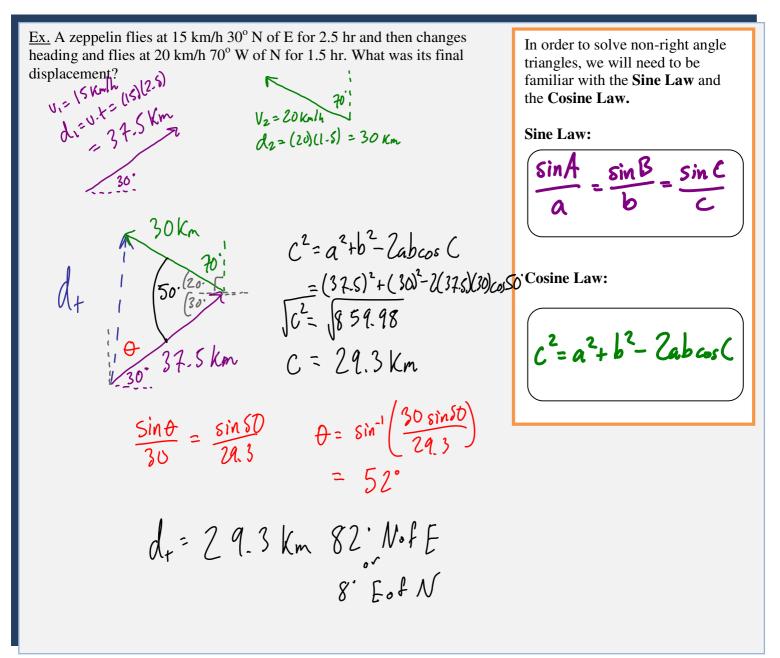


# <u>Vectors and Kinematics Notes</u> 4 – Vector Addition and Subtraction

overg,	SCALAR	VECTOR	Vector Addition	
	Speed	Velocity	Whenever we add vectors we use	
	true	displacement	tip to tail method	
	distance	acceleration	To find the total or resultant vector, simply draw	
	mass	force	an arrow from the start to	
	temperature	momentum	the finish	
\	When we draw vectors	we represent them as		
_	arrows			
Ex: A student in a canoe is trying to cross a 45 m wide river that flows due East at 2.0 m/s. The student can paddle at 3.2 m/s a. If he points due North and paddles how long will it take him to cross the river? $V_{y} = \frac{1}{2Y} + \frac{1}{2} = \frac{4y}{V_{y}} = \frac{45}{32mls} = \frac{145}{145}$ b. What is his total velocity relative to his starting point in part a? $V_{river} = 20 \text{ n/s}$ $V_{R}^{2} = V_{vast}^{2} + V_{river}^{2}$ $V_{le} = \sqrt{(3.2)^{2} + (2.0)^{2}} = 3.77 \text{ m/s} = \frac{3.8 \text{ n/s}}{145}$ $V_{lbsst} = 3.2 \text{ n/s}$ c. If he needs to end up directly North across the river from his starting point, what heading should he take? $V_{river} = 20 \text{ n/s}$ $S_{1A} \oplus = \frac{2.0}{3.2}$ $S_{1A}^{*} \oplus \frac{2.0}{3.2} = 32^{\circ} \text{ Uof N}$ $V_{river} = 20 \text{ n/s}$ $S_{1A} \oplus = \frac{2.0}{3.2}$ $S_{1A}^{*} \oplus \frac{2.0}{3.2} = 32^{\circ} \text{ Uof N}$				
d. ]	d. How long will it take him to cross the river at this heading? $V_{\text{bonf}}^{2} = V_{R}^{2} + V_{\text{river}}^{2} \qquad V_{R} = (V_{\text{bon}}^{2} - V_{\text{river}}^{2} = \int (3.2)^{2} - (2.2)^{2} = 2.50 \text{ m/s}$ $V_{y} = \frac{d_{y}}{f} \qquad f = \frac{d_{y}}{V_{y}} = \frac{45 \text{ m}}{2.50 \text{ m/s}} = \boxed{18 \text{ s}}$			

## Vector Addition - Trig Method

In the previous example we added perpendicular vectors which gave us a nice simple right triangle. In reality it's not always going to be that easy.



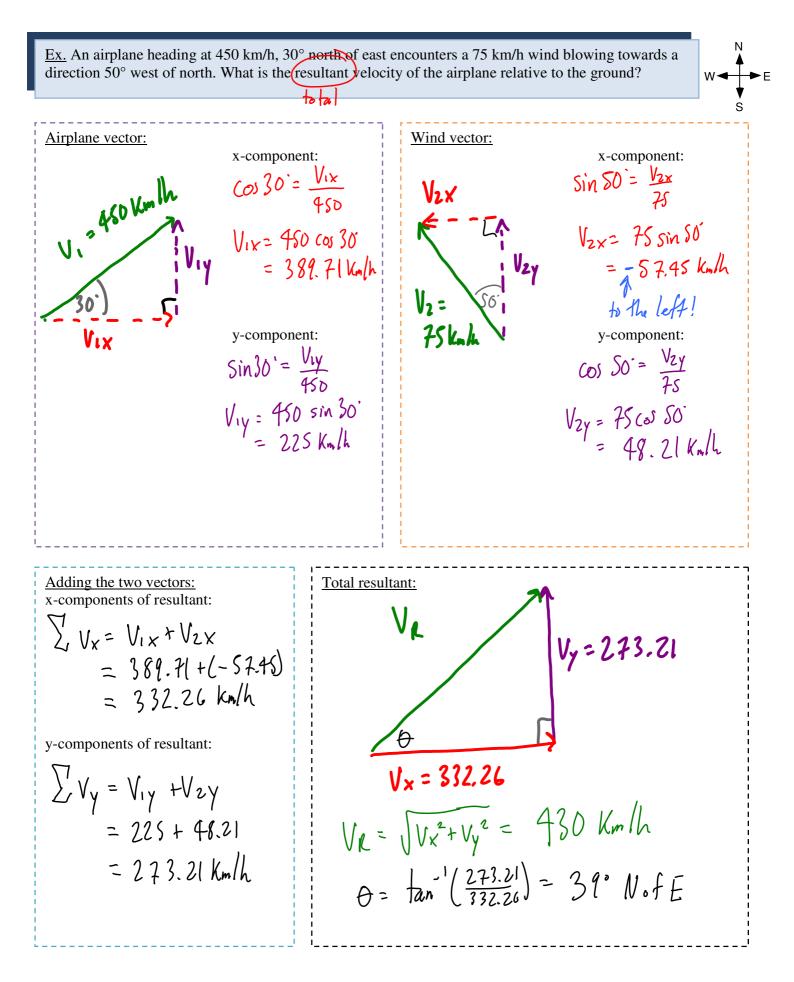
# Vector Addition – The Component Method

There is another method that we can use when adding vectors. This method is a very precise, stepwise approach, however it is the only way we can add 3 or more vectors.

- Draw each vector
- Resolve each vector into x and y components
- Find the **total sum** of x and y vectors
- Add the x and y vectors
- Solve using trig

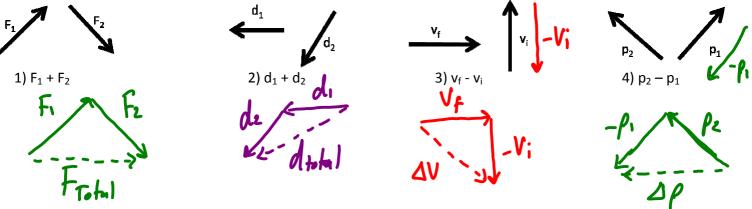
**REMEMBER:** When using x and y components...

• up and right are "+" • down and left are "-"

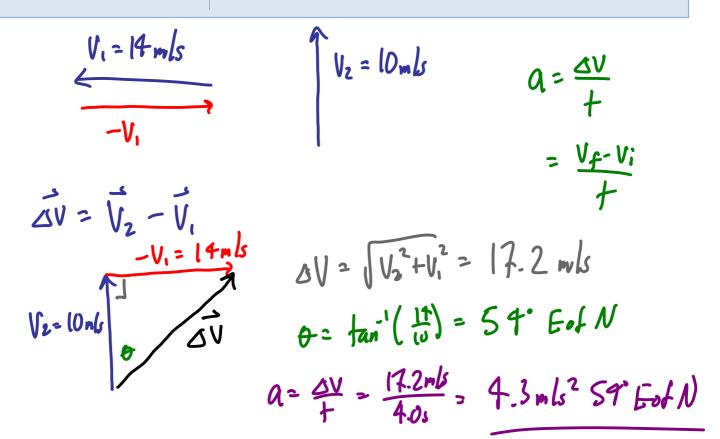


## Vector Subtraction

With vectors a negative sign indicates that... if points in the exact opposite direction When subtracting vectors we still draw them *tip to tail*, except... We reverse the negative vector We generally subtract vectors when dealing with a <u>Change</u> in a vector quantity. Recall: Change = final - initial  $\overrightarrow{U} = \overrightarrow{V_F} - \overrightarrow{V_1}$ Draw the Following



<u>Ex</u>: A cyclist is traveling at 14 m/s west when he turns due north and continues at 10 m/s. If it takes him 4.0 s to complete the turn what is the magnitude and direction of his acceleration?



Summary of Methods Say we want to add two vectors  $\vec{d}_1 + \vec{d}_2$ s d, d<sub>z</sub> You have 2 choices 2.) Component Method 1) <u>Trig Method</u> Just add them! d. dzy d total hhal dy total dx distal dix d<sub>zx</sub>

# Unit 2: Kinematics in 2D

4 - Projectile Motion Types 1 and 2 Remember that the x and y-components are <u>perpendicular</u> and therefore totally <u>independent</u>.

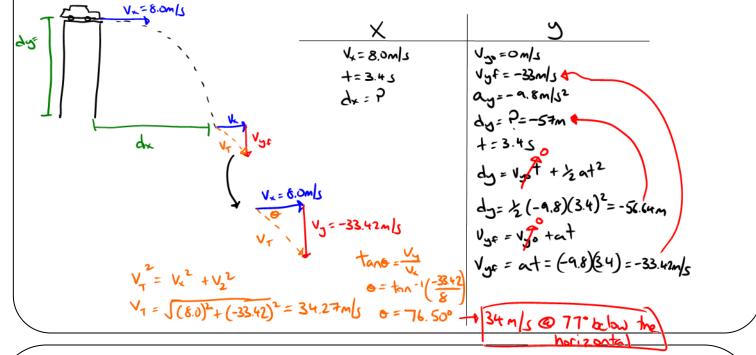
<u>X-components</u>	<u>Y-components</u>			
There is no <u>Net Force</u> working on the projectile in the X and the acceleration is always <u>Zero</u> . Therefore the only equation we can ever use is: $V_{4} = \frac{d_{4}}{t}$	In this case there is always a constant acceleration of <u>-9.8 m/s2</u> . Because of this we need to use the <u>Big Three</u> . $V_{f} = V_{0} t_{0} t_{1}$ . $V_{f} = V_{0} t_{0} t_{1}$ . $V_{f}^{2} = V_{0}^{2} + 2ad$			
The only value that can ever be used on both sides is <u>time</u> because it is a <u>scalar</u> . <u>Time is the "gatekeeper" of projective problems</u> <u>Problem Type 1:</u> A student sits on the roof of their house which is 12 m high. She can launch water-balloons from a slingshot at 14.0 m/s. If she fires a water-balloon directly horizontally:				
a. How long will it be airborne? This depends on: it's beight above the ground (dy) b. How far forward will it travel?				
b. How far forward will it travel? This depends on: it's horizontal relacity (Vx) or $V_x = 14m/s$ $V_x = 14m/s$ $d_x$ ? t = ?1.565s $d_x = V_x \cdot t$ $d_x = (44m/s)(1.565s)$ $d_x = 22m$	by the time its in the air(t) $V_{ys} = Om/s$ $U_{yF} =$ $d_y = -12m$ $a_y = -9.8m/s^2$ t = 1.65 $d = V_x f + \frac{1}{2}a + 2$ $t = \int \frac{2(-12)}{-a_s} = 1.565s$			

ط = 12

Example: A Cutlass Supreme drives straight out of a parking garage at 8.0 m/s and hits the water 3.4 s later.

a. How far did the car fall?

b. What was his total impact velocity? (magnitude and direction)



<u>Problem Type 2:</u> The Dukes of Hazzard are traveling at 85 km/h when they hit a jump that makes an angle of  $25^{\circ}$  above the horizontal.

X Y Q The dx = Vyf = omls a. How long are they airborne? Vx=21.40m/s Vys=9.978m/s b. How far forward do they fly through the air? += 2.0365 ay=-9.8 m/s2 c. What is their maximum height? 85km/hr = 23.61m/J only horizontal velocity (ve) b) d= vx t dy=  $t_{\frac{1}{2}} = 1.018s$ += 2.036s a)  $V_{yf} = V_{y0} + at_{\frac{1}{2}}$ dr=(21.40)(2.036)  $d_{+} = 43.57m$  $d_{+} = 44m$  $t_{\frac{1}{2}} = \frac{V_{yf} - V_{y}}{\alpha} = \frac{0 - 9.978}{-9.8}$  $t_{12} = 1.018s$  ; t = 2.03cg  $c^{0} (a + peak)$   $c^{0} V_{12} = V_{10}c^{2} + 2ady$ v = 23.61mls (Sin(25°))=9.978mls v Vx=23.61 m/s (cas(25))=21,40ms  $d_y = \frac{-V_{yo}^2}{2!} = \frac{-(9.978)^2}{2(-9.5)^2} = 5.1m$  <u>Example</u>: A quarterback launches a ball to his wide receiver by throwing it at 12.0 m/s at  $35^{\circ}$  above horizontal.

a. How far downfield is the receiver?

b. How high does the ball go?

c. At what other angle could the quarterback have thrown the ball and reached the same displacement?

$$\frac{x}{d_{x}} = \frac{x}{d_{y}} =$$

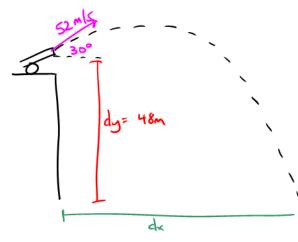
# Problem Type 3:

Ex: A cannon is perched on a 48 m high cliff. It aims 30° above the horizontal and fires a shell at 52 m/s. Find: a) How long it takes for the sheel to hit the ground.

dx=

= 304m

b) The distance it lands from the base of the cliff.



( vy= (52) sin(30) = 26.0m/s vx=(52)(25(30°)=45.0m/s

Х Vy= 26.0m/s dx= Vx=45.0m/s +=6.765  $V_{yf} = -48M$  $G_{y} = -9.8M$ b) dx = Vx 7 = (45.0)(6.76) += Vy = Vy + 2ad Vyf = + Vy2 + 2ad  $V_{y}F = \frac{1}{(2L)^{2} + 2(-9.8)(-48)}$ Vyf = +40.2 m/s regative volue! (go a)  $V_{yf} = V_{yo} + at$  $f = \frac{V_{yf} - V_{yo}}{\alpha} = \frac{(4\alpha 2) - (2\epsilon \cdot \alpha)}{-\alpha \epsilon}$ t=6.76s

Ex: A BMXer leaves a ramp traveling at 65 km/h at a trajectory of  $40^{\circ}$  above the horizontal. *After* reaching his max height he strikes the top of a building 5.8 m above the ground.

. .

- a) What is the horizontal distance from the ramp to the building?
- b) What is his speed when he hits the building?

$$\frac{1}{4} = \frac{1}{4} = \frac{1}$$

## WARNING: AP LEVEL QUESTION ALERT

Evel Knievel is in a cannon with an initial height of 1.00 meter above the ground. When lit, the cannon fire's Evel at a speed of 26.6 m/s at an angle of 50° above the horizontal. Evel travels a horizontal distance of 60.0 meters to a 10.0 meter high castle wall. (Assume air friction is negligible.)

- a. How long does it take Evel to reach a point directly about the wall?
- b. Determine Evel's height (above the ground) at the time he travels over the wall.
- c. Determine the velocity (magnitude and direction) of Evel as he passes over the castle wall.
- d. If an exact clone of Evel is fired with the same initial speed at an angle of 55.5° above the horizontal, will he clear the wall?

(a) 
$$t = \frac{d_x}{V_x} = \frac{60.0m}{17.1m/s} = 3.51s \text{ gate keeper} \frac{1}{d_x = 60.0m} \frac{1}{g_x = 9.5m/s}$$
  
 $V_x = 26.6 \cdot (245) V_{20} = 26.6 \sin(50)$   
 $V_x = 17.1m/s = V_{20} = 26.6 \sin(50)$   
 $V_x = 17.1m/s = V_{20} = 20.3m/s$   
 $d_y = (1.0) + (20.3)(3.51) + (0.5)(-9.8)(3.51)^2$   
 $d_y = 11.9m$   
(c)  $V_x$  is constant!  
 $V_{3f} = V_{30} + gt = (20.3m/s) + (-9.8m/s^2)(3.51s) = -114.1m/s$   
 $V_x = \sqrt{V_x^2 + V_{30}^2}$   $0 = t_{30} - \frac{1}{V_{20}}$   
 $V_r = \sqrt{V_x^2 + V_{30}^2}$   $0 = t_{30} - \frac{1}{V_{20}}$   
 $V_r = \sqrt{V_x^2 + V_{30}^2}$   $0 = t_{30} - \frac{1}{V_{20}}$   
 $V_r = 22.2m/s$   $0 = 39.5^{\circ}$   
 $122.2m/s$   $0 = 39.5^{\circ}$   
 $V_{30} = 21.9m/s \cdot \cos(55.5) = 15.1m/s$   $g = -9.8m/s$   
 $t = \frac{d_x}{V_x} = \frac{60.0m}{15.1m/s} = 3.98s$   
 $d_y = x_0 + V_{30} + 1 + 2gt^2$   
 $d_y = (1.0) + (21.9)(3.98) + (0.5)(-9.8)(3.98)^2$   
 $d_y = (0.5m)$  yes  $1 = 1$  if he is horizontal.