Rotational Dynamics

2 – Rotational Inertia, Angular Momentum and Kinetic Energy

In the previous section we explored the Kinematics of rotational motion, in this section we explore the dynamics of rotational motion.

Example:
If you open two doors with the same dimensions pushing with the same force, the first made of Balsa wood (160 kg/m^3) and the second made with Black Ironwood (the densest known wood 1355 kg/m^3), which door opens faster? Why?

balsa wood $\rightarrow$ less mass $\therefore F_{net} = ma$ it has a greater $\vec{a}$

Newton's Second Law: $\boxed{\sum F = ma = mr\alpha}$

$\alpha = \dfrac{a}{r}$

$a = \alpha \cdot r$

But remember in this unit we also explored torque! $\boxed{\tau = F \cdot d}$

If the force is applied to the edge of a rotating object then $d = r$.

$$\boxed{\tau = F \cdot r = mr^2\alpha}$$

Newton's First Law: States that an object in motion will stay in motion.

This is related to the inertia of the object.

Rotational Inertia (mr^2): effectively tells us the tendency for objects moving in a circle to keep moving in a circle.

If several objects are moving in a circle we must total sum of their Rotational Inertias to find the **Moment of Inertia (I).**

$$\boxed{I = \sum mr^2}$$

Units for moment of inertia are $kg \cdot m^2$.

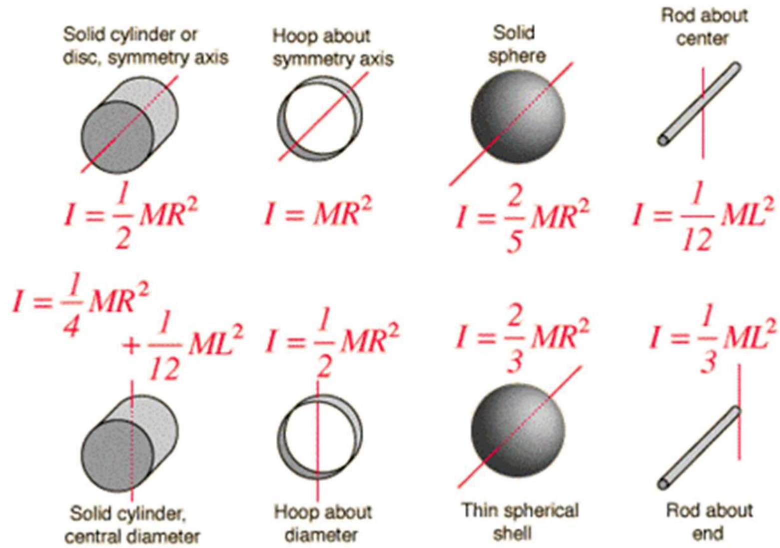
And because…

$$\sum \tau = \sum mr^2 \cdot \alpha$$

$$\sum \tau = I \cdot \alpha$$

Mass Distribution

Different shapes have different mass distributions and as a result have different moments of inertia.



Example:
A bicycle rim has a diameter of 0.65 m and a moment of inertia (measured about its center) of $0.19\ kg{\cdot}m^2$. What is the mass of the rim? ring

$I = 0.19\ kg \cdot m^2 \quad I = mr^2$

$d = 0.65$

$$m = \frac{I}{r^2} = \frac{0.19}{(0.325)^2} = \boxed{1.8\ kg}$$

Example:
Find the net torque required for your hip muscles to swing your leg at an angular acceleration of 5.0 rad/s^2, if you assume the leg is a solid rod with mass = 20 kg and length of 0.90 m. $I = \frac{1}{3}mL^2$

$$\Sigma\tau = I\alpha$$
$$= \frac{1}{3}mL^2\alpha = \frac{1}{3}(20)(0.90)^2(5.0)$$
$$= 40.5 = \boxed{41\ N{\cdot}m}$$

Example:
Find the net torque required to accelerate a DVD from rest to its operating speed of 4.0 rad/s, in 2.0 seconds, if the DVD is 52 grams and has a diameter of 20. cm. $I = \frac{1}{2}mr^2$

$$\tau = I\alpha$$

Example:
A playground merry-go-round starts at rest and is accelerated uniformly, completing 4.00 rotations in 6.00 s.

(a) Calculate its angular acceleration. $4.00\ \text{rotations} \times \frac{2\pi}{1\ \text{rotation}} = 25.13$

ω
$\omega_0 = 0$
$\alpha = ?$
$\theta = 25.13\ rad$
$t = 6.00\ s$

$$\theta = \cancel{\omega_0 t}^{\,0} + \frac{1}{2}\alpha t^2$$
$$\alpha = \frac{2\theta}{t^2} = \frac{2(25.13)}{(6.00)^2} = 1.396 = \boxed{1.4\ rad/s^2}$$

(b) If the merry-go-round is disk-shaped, with a mass of 115 kg and a radius of 1.8 m, calculate the net torque acting on the merry-go-round. $I_{Disc} = \frac{1}{2}mr^2$

$$\Sigma\tau = I\alpha = \frac{1}{2}(115)(1.8)^2(1.396)$$
$$= \boxed{260\ N{\cdot}m}$$

In this unit we have connected our understanding of Linear Kinematics/Dynamics to Rotational Kinematics/Dynamics. ***Well why stop there!?***

Name:	**Linear Momentum**	**Angular Momentum**
Symbol:	p	L
Formula:	$p = mv$	$L = I\omega$ ($I \to kg \cdot m^2$, $\omega \to \frac{rad}{s}$)
Units	$kg\ m/s$	$kg \cdot m^2/s$

Name:	**Linear Impulse**	**Angular Impulse**
Symbol:	Δp	ΔL
Formula:	$\Delta p = m\Delta v$ $\Delta p = F_{net} \cdot t$	$\Delta L = \tau \cdot t$
Units:	$kg\ m/s$	$kg \cdot m^2/s$

Name:	**Linear Kinetic Energy**	**Rotational Kinetic Energy**
Symbol:	E_K	E_R
Formula:	$E_K = \frac{1}{2}mv^2$	$E_R = \frac{1}{2}I\omega^2$ ($I \to kg \cdot m^2$, $\omega^2 \to (\frac{rad}{s})^2$)
Units:	J ($kg\ m^2/s^2$)	J ($kg\, m^2/s^2$)

<u>Example</u>:
Suppose that a figure skater has a moment of inertia of 6.5 kg m^2 when her arms are outstretched and 3.8 kg m^2 when her arms are pulled in. She is initially spinning with an angular velocity of 8.2 rad/s with her arms outstretched and then pulls her arms in. What is her final angular velocity?

$L_i = L_f$ ← conservation of angular momentum

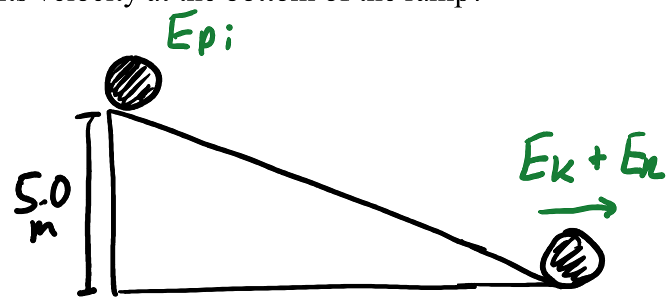
$I_i \omega_i = I_f \omega_f$ $\quad \omega_f = \frac{I_i \omega_i}{I_f} = \frac{(6.5)(8.2)}{3.8} = 14$ rad/s

How does her does her initial rotational energy compare to her final rotational energy?

$E_i < E_f$ $\quad E_R = \frac{1}{2} I \omega^2$

Example:

A ball is rolled down a ramp with a height of 5.0 m. What is its velocity at the bottom of the ramp?



Assumptions:

- The ball acts as a…solid sphere.
- The ball rolls without…slipping.

$$E_p = E_K + E_R$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$\omega = \frac{v}{r}$

$I_{sphere} = \frac{2}{5}mr^2$

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 + \frac{1}{2}\left(\frac{2}{5}\cancel{m}\cancel{r^2}\right)\left(\frac{v}{\cancel{r}}\right)^2$$

$$gh = \frac{1}{2}v^2 + \frac{1}{5}v^2$$

$$= \frac{5}{10}v^2 + \frac{2}{10}v^2$$

$$gh = \frac{7}{10}v^2$$

$$v = \sqrt{\frac{10}{7}gh}$$

$$= \sqrt{\frac{10}{7}(9.8)(5.0)}$$

$$= \boxed{8.4\ m/s}$$