Rotational Dynamics
1 – Rotational Kinematics

In **Kinematics**, we studied motion along a straight line and introduced such concepts as displacement, velocity, and acceleration. **Two-Dimensional Kinematics** dealt with motion in two dimensions. **Projectile motion** is a special case of **two-dimensional kinematics** in which the object is projected into the air, while being subject to the gravitational force, and lands a distance away. In the next few sections, we consider situations where the object does not land but moves in a curve.

When objects rotate about some axis, for example, when a CD (compact disc) rotates about its center each point in the object follows a circular arc.

The rotational angle is the amount of rotation and is analogous to linear distance. We define the rotation angle θ to be the ratio of the arc length to the radius of curvature.

$$\Delta\theta = \frac{\Delta s}{r}$$

Where: $\Delta\theta$ = rotational angle
Δs = arc length
r = radius

2π = 1 revolution

$v = \frac{d}{t}$

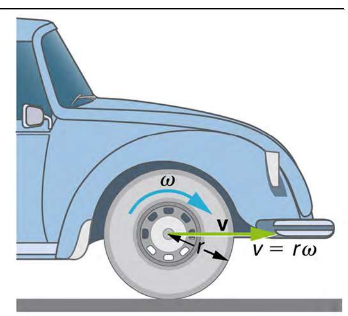
Angular Velocity (ω): A measure of the rate of change of the rotational angle.

$$\omega = \frac{\Delta\theta}{t} = \frac{\Delta s}{r\,t} = \frac{v}{r}$$

Note: Units for Angular Velocity are rad/s.

Example:
Calculate the angular velocity of a 0.300 m radius car tire when the car travels at about 54 km/h. $\div 3.6 = 15$ m/s

$$\omega = \frac{v}{r} = \frac{15\text{ m/s}}{0.300\text{ m}} = 50\text{ rad/s}$$



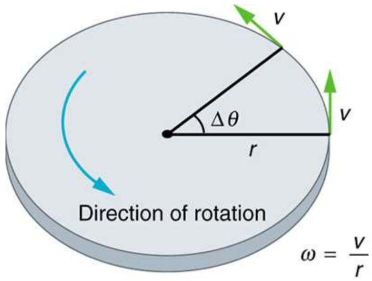
What would the angular velocity be for a car traveling at the same speed that had tires which were four times larger?

$\omega = \frac{v}{r}$ $\quad \omega \propto \frac{1}{r}$

$\omega_1 = \frac{v_1}{r_1} = 50$

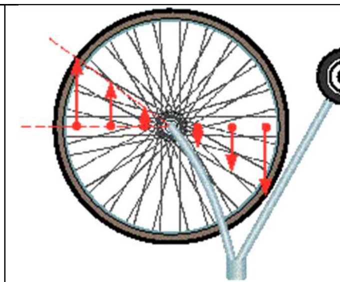
$\omega_2 = \frac{v_1}{4r_1} = \frac{50}{4} = 12.5$ rad/s

Angular Acceleration (α): the rate of change of angular velocity.



$$\alpha = \frac{\Delta\omega}{t}$$

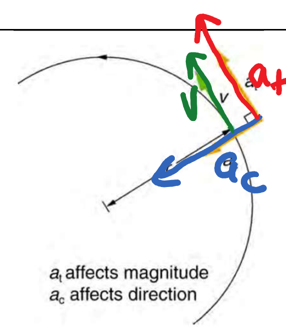
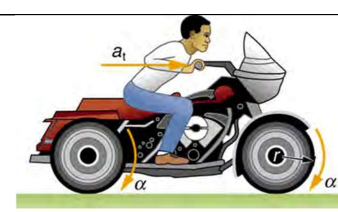
units: rad/s^2

<u>Example</u>: Suppose a Rockridge student puts their bicycle on a stand and starts the rear wheel spinning from rest to a final angular velocity of 250 rpm (*revolutions per minute*) in 5.00 s. (a) Calculate the angular acceleration in rad/s^2. (*Hint: how many rad's are there in 1 revolution?*) $250 \frac{\cancel{rev}}{\cancel{min}} \times \frac{2\pi}{\cancel{rev}} \times \frac{1 \cancel{min}}{60 s} = 26.18$ rad/s $\alpha = \frac{\Delta\omega}{t} = \frac{(26.18-0)}{5.00} = 5.236 = \boxed{5.2 \text{ rad/s}^2}$	

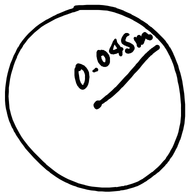
(b) If they now slam on the brakes, causing an angular acceleration of – 87.3 rad/s^2, how long does it take the wheel to stop?

$\alpha = \frac{\Delta\omega}{t}$ $\quad t = \frac{\Delta\omega}{\alpha} = \frac{(0-26.18)}{-87.3} = \boxed{0.30 s}$

But what about the connection between circular motion and linear motion?

If an object experiences an angular acceleration then the velocity of any point on the object must be increasing or decreasing The linear acceleration is tangent to the path of rotation and therefore we use a_t. linear acceleration $\quad$ linear velocity $(v=\omega r)$ $a_t = \frac{\Delta v}{t} = \boxed{\frac{\Delta\omega r}{t}} = \alpha r$ $\alpha = \frac{\Delta\omega}{t}$ $$\alpha = \frac{a_t}{r}$$	
<u>Example</u>: $\div 3.6 = 30$ m/s A powerful motorcycle can accelerate from 0 to 108 km/h in 4.20 s. What is the angular acceleration of its 0.320 m radius wheels? $a = \frac{\Delta v}{t} = \frac{(30-0)}{4.20} = 7.143$ m/s^2 $\quad \alpha = \frac{a_t}{r} = \frac{7.143}{0.320}$ $= 22.3$ rad/s^2	

The good news is our understanding of linear kinematics allows us to explore rotational kinematics.



Rotational	Translational
$\theta = \bar{\omega} t$	$x = \bar{v} t$
$\omega = \omega_0 + \alpha t$	$v = v_0 + at$
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	$x = v_0 t + \frac{1}{2}at^2$
$\omega^2 = \omega_0{}^2 + 2\alpha\theta$	$v^2 = v_0{}^2 + 2ax$

<u>Example</u>: A deep-sea fisherman hooks a big fish that swims away from the boat pulling the fishing line from his fishing reel. The whole system is initially at rest and the fishing line unwinds from the reel at a radius of 4.50 cm from its axis of rotation. The reel is given an angular acceleration of 110 rad/s^2 for 2.00 s.

a) What is the final angular velocity of the reel?

$\omega =$

$\omega_0 = 0$

$\alpha = 110$

$\theta =$

$t = 2.00$

$$\omega = \omega_0 + \alpha t = 0 + (110)(2.00) = 220 \text{ rad/s}$$

b) At what speed is fishing line leaving the reel after 2.00 s elapses?

$$\omega = \frac{v}{r}$$

$$v = \omega \cdot r = (220)(0.045) = \boxed{9.9 \text{ m/s}}$$

c) How many revolutions does the reel make?

$$\theta = \cancel{\omega_0}^{\,0} t + \frac{1}{2}\alpha t^2 = \frac{1}{2}(110)(2.00)^2 = 220 \text{ rad} \times \frac{1 \text{ rev}}{2\pi} = \boxed{35 \text{ rev}}$$

1 rev = 2π rad

d) How many meters of fishing line come off the reel in this time?

$$\theta = \frac{\Delta s}{r}$$

$$\Delta s = \theta \cdot r = (220)(0.045) = \boxed{9.9 \text{ m}}$$