Electric Circuits Notes

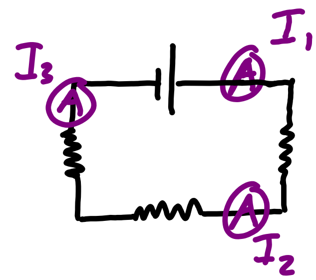
3 – Kirchhoff's Laws

We have already seen that we can connect devices to a circuit in two ways: series or parallel.

The manner in which we attach components of a circuit can greatly affect the nature of the circuit in particular its resistance there are a number of laws that we must use called: Kirchoff's Laws!

Kirchhoff's Current Law

For a series circuit:

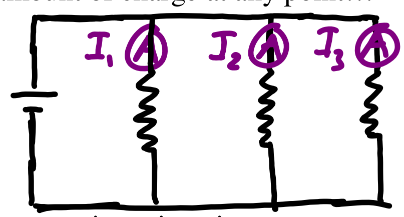


In a series circuit there is only one path so the current must be…the same everywhere! (What would happen if it wasn't?)

$$I_T = I_1 = I_2 = I_3$$

For a parallel circuit:

In a parallel circuit the charge can take different paths. Therefore the amount of ~~charge at any point~~… current flowing through each pathway must equal the total



$$I_T = I_1 + I_2 + I_3$$

Kirchhoff's Current Law can be directly stated as: the sum of currents entering a junction… must equal the sum of currents leaving a junction.

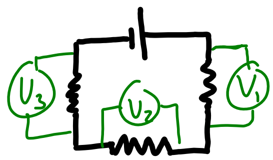
Kirchhoff's Voltage Law

Kirchhoff's Voltage Law is stated as: The sum of the potential differences in a circuit must… add up to zero

In a way this is simply restating the…Law of Conservation of Energy!

Remember that there is an increase in the potential across the terminals of a cell and that there is a decrease in potential across a resistor. Essentially these increases and drops must add up to zero.

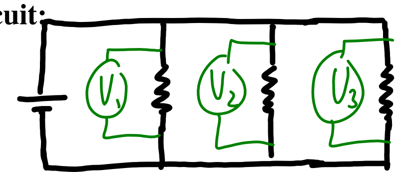
For a series circuit:



$$V_T = V_1 + V_2 + V_3$$

Since there is only one path, the total voltage increase across the battery must equal the total drop across each resistor.

For a parallel circuit:



$V_T = V_1 = V_2 = V_3$

Note that the potential difference is… the same across each resistor. (Why is that?)

Kirchhoff vs Ohm

Kirchhoff does not have a law for resistance. However we can perform an arduous derivation to find the formula using Kirchhoff's other law and Ohm's Law.

Instead, let's just reason it out.

For a series circuit: as more resistors are added, total resistance increases

$R_T = R_1 + R_2 + R_3$

The total resistance in a series circuit is the sum of all the resistors. Since each electron must push its way through each resistor, it should make sense that the resistances are cumulative.

For a parallel circuit:
as more resistors are added in parallel, the total resistance decreases.

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

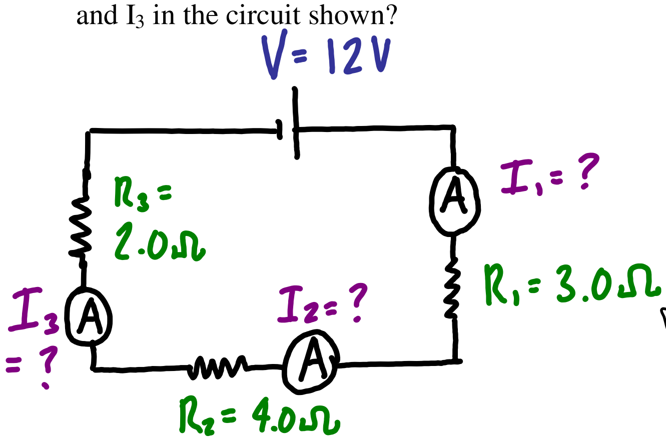
We already know that as we add resistors in parallel, the total resistance… decreases!

If our marching soldiers are forced through one path, then there will be much more friction than if there are multiple paths to choose from. This is true even if the additional pathways are of **higher resistance**.

Let's recap:

Formula	Series	Parallel
V	$V_T = V_1 + V_2 + V_3$	$V_T = V_1 = V_2 = V_3$
I	$I_T = I_1 = I_2 = I_3$	$I_T = I_1 + I_2 + I_3$
R	$R_T = R_1 + R_2 + R_3$	$R_T^{-1} = R_1^{-1} + R_2^{-1} + R_3^{-1}$

<u>Example:</u>
What are the values of I_1, I_2 and I_3 in the circuit shown?

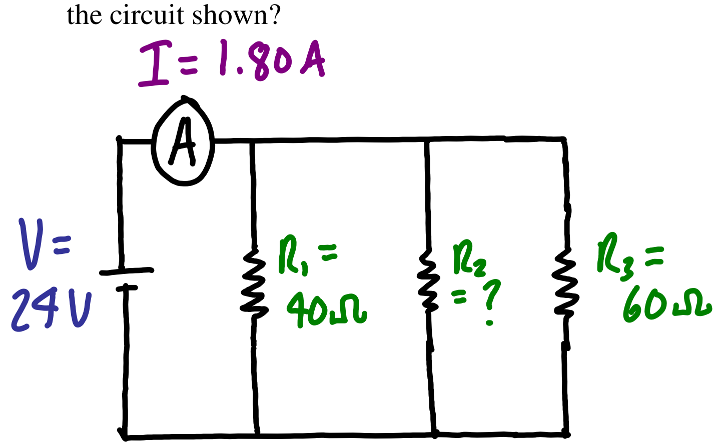


$$R_T = R_1 + R_2 + R_3$$
$$= 3.0 + 4.0 + 2.0$$
$$= 9.0\,\Omega$$

$$I_T = \frac{V_T}{R_T} = \frac{12\,V}{9.0\,\Omega} = 1.33\,A$$

$$\boxed{I_T = I_1 = I_2 = I_3 = 1.33\,A}$$

<u>Example:</u>
What is the value of R_2 in the circuit shown?



There is another way that doesn't use R_T ... can you find it?

$$R_T = \frac{V_T}{I_T} = \frac{24\,V}{1.80\,A} = 13.333\,\Omega$$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

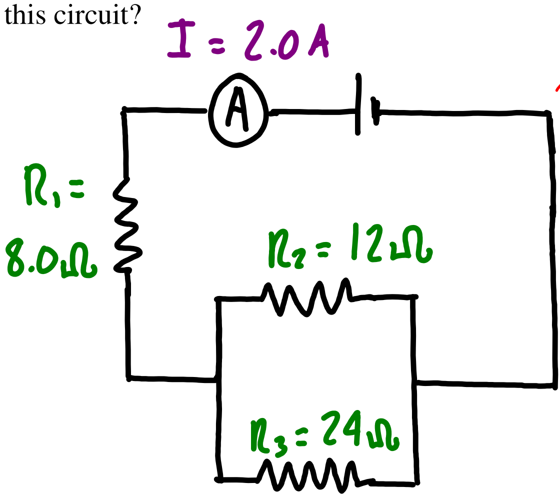
$$\frac{1}{R_2} = \frac{1}{R_T} - \frac{1}{R_1} - \frac{1}{R_2}$$

$$\frac{1}{R_2} = \frac{1}{13.333} - \frac{1}{40} - \frac{1}{60}$$

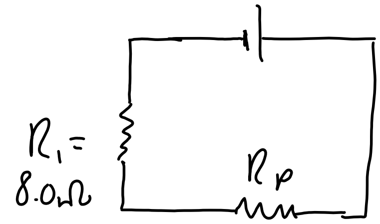
$$\frac{1}{R_2} = 0.03333$$

$$\boxed{R_2 = 30\,\Omega}$$

Example:
What is the potential difference supplied by the power source in this circuit?



One strategy is to treat the parrallel resistor as a single resistor:



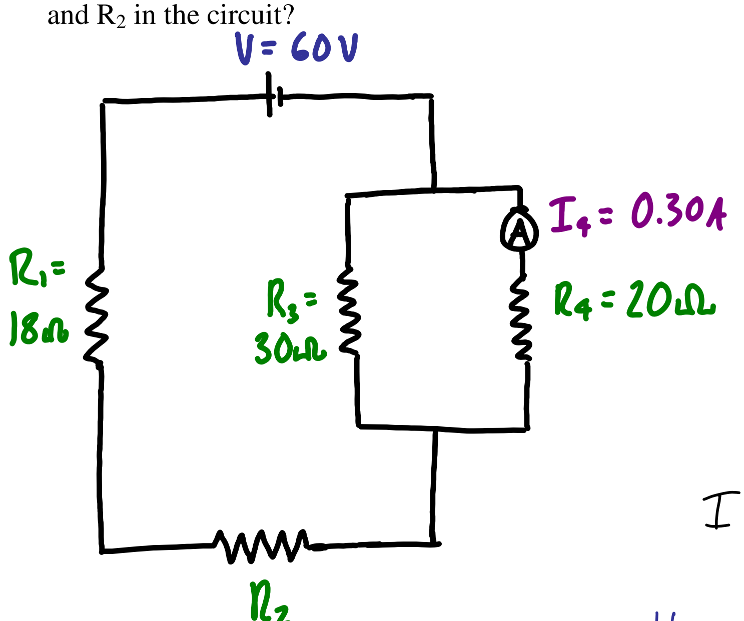
$$\frac{1}{R_p} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{12} + \frac{1}{24}$$

$$R_p = 8.0\,\Omega$$

$$R_T = R_1 + R_p = 8.0 + 8.0 = 16.0\,\Omega$$

$$V_T = I_T R_T = (2.0\,A)(16.0\,\Omega)$$
$$= \boxed{32\,V}$$

What are the values of V_1, V_2 and R_2 in the circuit?



$$V_4 = I_4 R_4 = (0.30A)(20\,\Omega)$$
$$= 6\,V$$

$$V_3 = V_4 = 6\,V$$

$$I_3 = \frac{V_3}{R_3} = \frac{6\,V}{30\,\Omega} = 0.20\,A$$

$$I_T = I_3 + I_4 = 0.30 + 0.20 = 0.50A$$

$$V_1 = I_1 R_1 = (0.5A)(18\,\Omega) = \boxed{9\,V}$$

$$V_T = V_1 + V_2 + V_p \leftarrow \text{parallel component}$$

$$V_2 = V_T - V_1 - V_p = 60 - 9 - 6$$
$$= \boxed{45\,V}$$

$$R_2 = \frac{V_2}{I_2} = \frac{45V}{0.50A} = \boxed{90\,\Omega}$$