Work, Energy and Momentum Notes

1 – Work and Energy

Work is defined as the **transfer** of energy from one body to another.
Or more rigorously:

$W = \Delta E$

We can calculate the work done on an object with:

$W = F \cdot d$

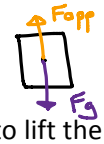
the units of work are Nm or Joules

Note that these are the same units as **torque** yet their values are used to describe *very* different quantities.

distance to pivot

Example 1 - Work against Gravity

How much work is required to lift a 2.0 kg textbook from the floor to a height of 1.5 m at a constant velocity?



Note:
W = Fd, but what force do we need to exert to lift the book at a constant velocity?

$W = F_g \cdot d = mgd$

$F_{app} = F_g$

$W = (2.0)(9.8)(1.5) = 29.4\,J$

Since the velocity is constant what is the net force acting on the book?

$F_{net} = F_{app} - F_g = 0$

Example 2 - Work on an object

How much work is done on a 4.0 kg medicine ball that is held at a height of 1.8 m for 10 s?

Note:
Is energy being used to hold the ball in this position?

YES!

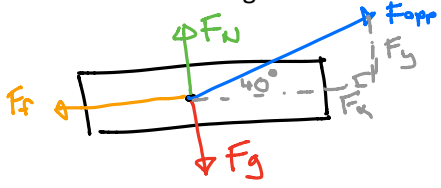
Is work actually being done ON THE BALL?

No! $d = 0\,m$

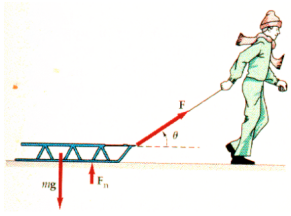
Example 3 – Forces at an angle

A plucky youngster is pulling his sled at a constant velocity of 1.2 m/s. He pulls the 15 kg sled with a force of 35 N at an angle of 40° to the horizontal. How much work does he do in pulling the sled 20 m?

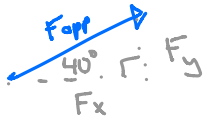
Draw an FBD showing the forces at work on the sled.



$W = F_x \cdot d$
$= F \cdot \cos(40)\, d$
$= 35 \cdot \cos(40)(20)$
$= 536\,J$



Break F_{boy} into its vertical and horizontal components. Does the vertical component of the force **do** any work?



No!

Rule: When finding the work done on an object we only consider…….

Component of force _____ in the direction of motion

Fancy New Definition of Work

$W = F_x \cdot d_x$

Example 4 – F_{net} vs. F_{app} Remember Work is defined by what force you apply on the system!

A biology student is pushing a rope 15 m across a flat surface. The student pushes the rope with a force of 220 N while the force of friction is 120 N. How much work is the student doing?

Note: To find the amount of work done by the student should we used F_{net} or F_{app}?

$$W = F_{app} \cdot d = (220N)(15\,m) = 3300J$$

Rule: When finding the **total** work done on an object we always use: F_{app}!

Example 5 – To scalar or not to scalar?

Work is the product of a scalar and a vector, but work is a Scalar. However work can be positive (+) or negative (−).

1) Car accelerates from rest 2) Skater stops suddenly 3) Lifting a object above your head 4) Ball rolls down a ramp

\+ − \+ \+

Types of Energy:

There are many forms of energy: mechanical, thermal, electrical, nuclear, chemical etc. One form can be converted into another by doing work.

In this chapter will be concerned mostly with potential and kinetic (and just a hint of thermal) energy.

Potential Energy (E_p):

Stored Energy

$E_p = mgh$

Remember:
Potential energy is always... relative to a reference point

Cause math is fun!
Deriving the E_p formula...

$E_p = Fd$
(in this case $F = F_g = mg$)

$E_p = mgd$
(let's change letters just for fun, and call h the vertical displacement)

$E_p = mgh$

HOORAY!

Kinetic Energy (E_k):

Energy of motion

$E_k = \frac{1}{2}mv^2$ 1

Cause math is fun!
Deriving the E_k formula...

$v^2 = v_o^2 + 2ad$
(take $v_o = 0$)
$v^2 = 2ad$
(but $a = F/m$)
$v^2 = \frac{2Fd}{m}$
(but $Fd = W = E_k$, when $v_o = 0$)

$v^2 = \frac{2E_k}{m}$

$E_k = \frac{1}{2}mv^2$

A portion of the total work

Example 6 - Work-Energy Theorem for Net Force

It is worth noting that the work done by the net force on an object is equal to the change in its kinetic energy:

$W = F_{net} \cdot d$

$W = m \cdot a \cdot d$

$V^2 = V_0^2 + 2ad$

$ad = \frac{1}{2}(V^2 - V_0^2)$

$W = \frac{1}{2}m(V^2 - V_0^2)$

$W = \frac{1}{2}m\Delta V^2 = \Delta E_k = F_{net} \cdot d$

Example:
A 1270 kg car accelerates from 15 m/s to 25 m/s over a distance of 75 m. Determine the average net force that was required to do this.

$\Delta E_k = F_{net} \cdot d$

$F_{net} = \frac{\Delta E_k}{d} = \frac{\frac{1}{2}m\Delta V^2}{d}$

$F_{net} = \frac{\frac{1}{2}(1270)(25^2 - 15^2)}{(75)} = 3390\,N$

Example:
The graph shows a variable force working on a 15.0 kg mass on a level surface which is initially at rest. Find:
a. The total amount of work done.
b. The final speed of the object assuming friction is negligible.

Area 1 = 20J
Area 2 = 80J
Area 3 = 40J

a) $W_T = 140J$

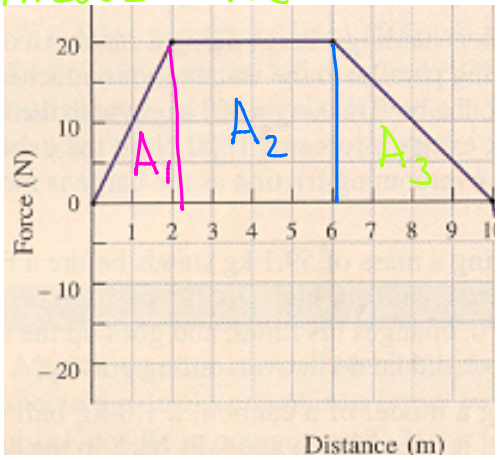
b) $W = \Delta E_k$

$W = E_{k_f} - E_{k_i}$

$W = \frac{1}{2}mv^2$

$V = \sqrt{\frac{2W}{m}}$

$V = 4.3\,m/s$



To summarize our work energy relationships:

$W_{Total} = \Delta E_T = F_{app} \cdot d$

$\Delta E_k = F_{net} \cdot d$

heat!

$\Delta E_H = F_f \cdot d$

The Law of Conservation of Energy

Energy cannot be Created or destroyed, only changed from one form into another. Therefore if only conservative forces act in in a system the total change in energy is always Zero!.

Total Initial Energy = Total Final Energy

$E_{ki} + E_{pi} = E_{kf} + E_{pf}$

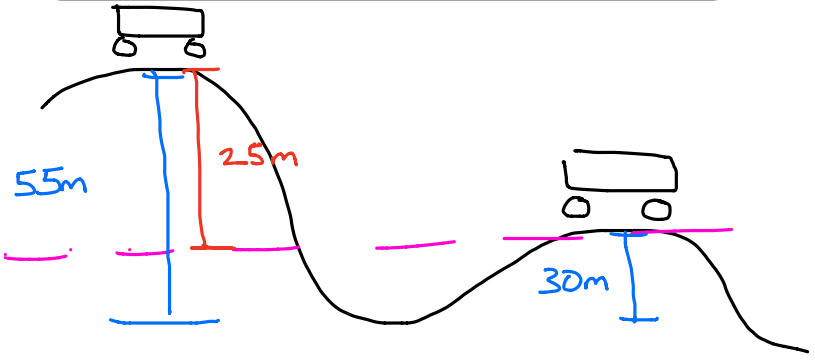
$\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f$

Total Change in Energy = 0

$\Delta E_k + \Delta E_p = 0$

$\Delta E_k = -\Delta E_p$

Example
The first peak of a roller coaster is 55 m above the ground. The 1200kg car starts from rest and goes down the hill and up the second hill which is 30 m high. How fast is the car traveling at the top of the second hill?



$E_{ki} + E_{pi} = E_{kf} + E_{pf}$

$E_{pi} = E_{kf}$

$mgh_i = \frac{1}{2}mv_f^2$

$v_f = \sqrt{2gh_i} = \sqrt{2(9.8)(25)}$

$= 22.1\,m/s$