Kinematics

And yet it moves.

—Galileo Galilei

Galileo, the “father of modern physics,” stated a distinction between the cause of motion and the description of motion. Kinematics is that modern description of motion that answers questions such as:

How far does this object travel?

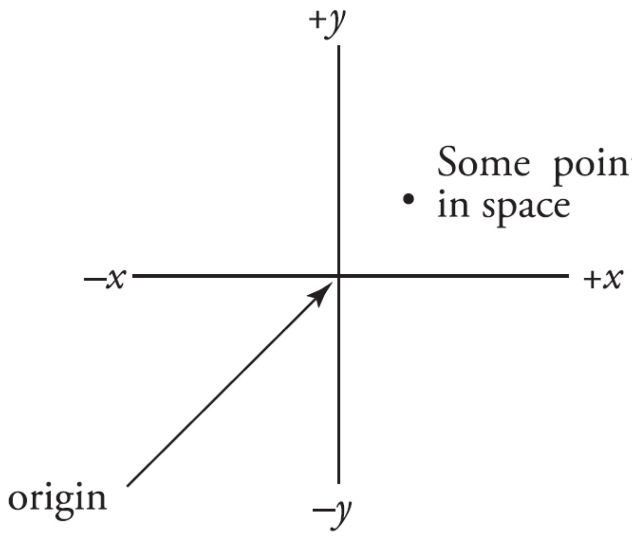
How fast and in what direction does it move?

At what rate does its speed change?

Kinematics are the mathematical tools for describing motion in terms of displacement, velocity, and acceleration.

POSITION

An object's position is its location in a certain space. Since it is difficult to describe an object's location, in mathematics, we typically use a coordinate system to show where an object is located. Using a coordinate system with an origin also helps to determine positive and negative positions. Typically we set the object in question as the origin and relate its surroundings to the object.



DISPLACEMENT

Displacement is an object's change in position. It's the vector that points from the object's initial position to its final position, regardless of the path actually taken. Since displacement means *change in position*, it is generically denoted Δs, where Δ denotes *change in* and *s* means spatial location. The displacement is only the distance from an object's final position and initial position. The **total distance** takes into account the total path taken (meaning if an object went backward and then forward, those distances are included in the total distance). Since a distance is being measured, the SI unit for displacement is the meter $[\Delta s] = \text{m}$.

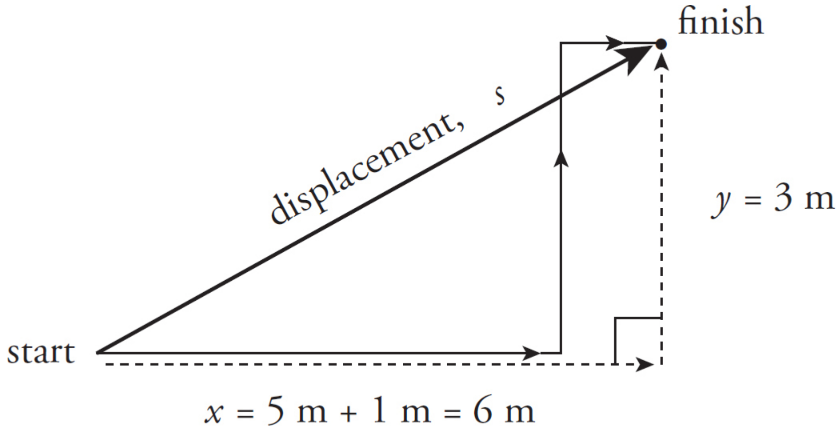
> **Pay Attention to What the Question Is Asking**
> Asking for displacement and total distance will give different answers for a problem.

> **Example 1** A rock is thrown straight upward from the edge of a 30 m cliff, rising 10 m, and then falling all the way down to the base of the cliff. Find the rock's displacement.

Solution. Displacement only refers to the object's initial position and final position, not the details of its journey. Since the rock started on the edge of the cliff and ended up on the ground 30 m below, its displacement is 30 m, downward. Its distance traveled is 50 m; 10 m on the way up and 40 m on the way down.

> **Example 2** An infant crawls 5 m east, then 3 m north, and then 1 m east. Find the magnitude of the infant's displacement.

Solution. Although the infant crawled a *total* distance of 5 + 3 + 1 = 9 m, this is not the displacement, which is merely the *net* distance traveled.



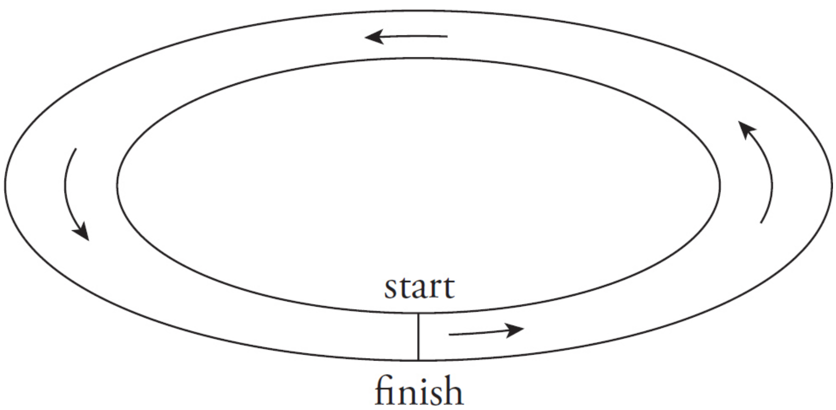
Math on the AP Physics 1 Exam
You must be proficient in basic trigonometry to do well on this test, so revisit that subject matter now if you need a refresher.

Using the Pythagorean Theorem, we can calculate that the magnitude of the displacement is

$$\Delta s = \sqrt{(\Delta x)^2 + (\Delta(y))^2} = \sqrt{(6\text{ m})^2 + (3\text{ m})^2} = \sqrt{45\text{ m}^2} = 6.7\text{ m}$$

Example 3 In a track-and-field event, an athlete runs exactly once around an oval track, a total distance of 500 m. Find the runner's displacement for the race.

Solution. If the runner returns to the same position from which she left, then her displacement is zero.



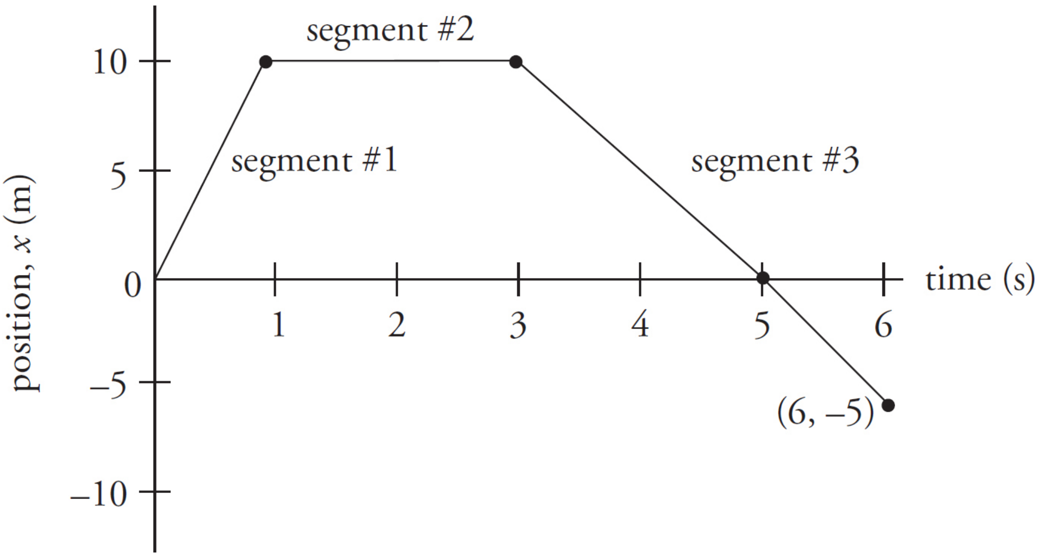
The *total* distance covered is 500 m, but the net distance—the displacement—is 0.

A Note About Notation

Δs is a more general term that works in any direction in space. The term x or "$\Delta x = x_f - x_i$" has a specific meaning that is defined in the x direction. However, to be consistent with AP notation and to avoid confusion between spacial location and speed, from this point on we will use x in our development of the concepts of speed, velocity, and acceleration. x is just a reference and can refer to any direction in terms of the object's motion (for example straight down or straight up).

LOOKING AT DISTANCE-VERSUS-TIME GRAPHS

You should also be able to handle kinematics questions in which information is given graphically. One of the popular graphs in kinematics is the position-versus-time (also referred to as a *p*-versus-*t*) graph. For example, consider an object that's moving along an axis in such a way that its position x as a function of time t is given by the following position-versus-time graph:



Geometry!
You will have to brush up on your coordinate geometry: slope and basic area of geometric shapes.

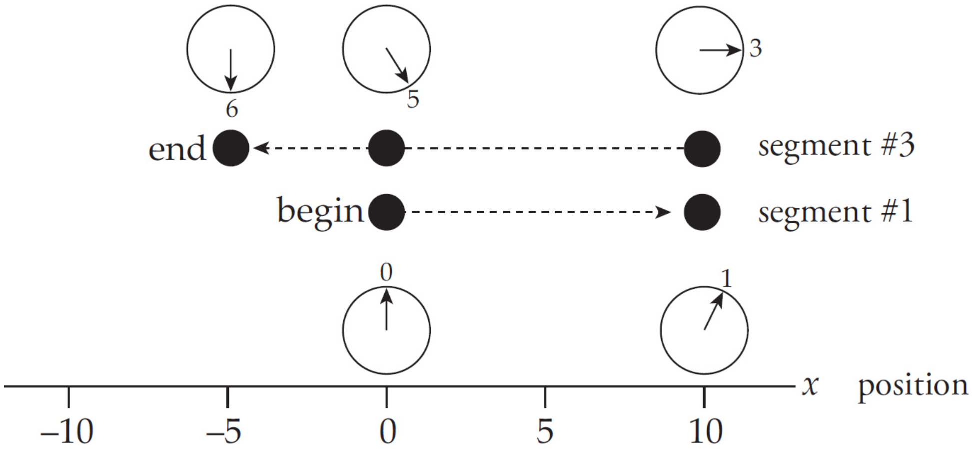
$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

What does this graph tell us? It says that at time t = 0, the object was at position x = 0. Then, in the first second, its position changed from x = 0 to x = 10 m. Then, at time t = 1 s to 3 s, it stopped (that is, it stayed 10 m away from wherever it started). From t = 3 s to t = 6 s, it reversed direction, reaching x = 0 at time t = 5 s, and continued, reaching position x = −5 m at time t = 6 s.

Example 4 What is the total displacement?

Solution. Displacement is just the initial position subtracted from the final position. In this case, that's −5 − 0 = −5 m (or 5 meters to the left, or negative direction).



What is the total distance traveled?

10 m	+	15 m	=	25 meters
segment #1		segment #3		

SPEED AND VELOCITY

When we're in a moving car, the speedometer tells us how fast we're going; it gives us our speed. But what does it mean to have a speed of say, 10 m/s? It means that we're covering a distance of 10 meters every second. By definition, **average speed** is the ratio of the total distance traveled to the time required to cover that distance:

$$\text{average speed} = \frac{\text{total distance}}{\text{time}}$$

The car's speedometer doesn't care what direction the car is moving. You could be driving north, south, east, west, whatever; the speedometer would make no distinction. *55 miles per hour, north*and *55 miles per hour, east* register the same on the speedometer: 55 miles per hour. Remember that speed is scalar, so it does not take into account the direction.

However, we will also need to include *direction* in our descriptions of motion. We just learned about displacement, which takes both distance (net distance) and direction into account. The single concept that embodies both speed and direction is called **velocity,** and the definition of average velocity is

$$\text{average velocity} = \frac{\text{displacement}}{\text{time}}$$

$$\bar{\mathbf{v}} = \frac{\Delta \mathbf{x}}{\Delta t}$$

units = meter/seconds

Note that the bar over the **v** means *average*. Because $\Delta\mathbf{x}$ is a vector, $\bar{\mathbf{v}}$ is also a vector, and because Δt is a *positive* scalar, the direction of $\bar{\mathbf{v}}$ is the same as the direction of $\Delta\mathbf{x}$**.** The magnitude of the velocity vector is called the object's **speed,** and is expressed in units of meters per second (m/s).

Note the distinction between speed and velocity. *Velocity is speed plus direction.* An important note: the magnitude of the velocity is the speed. The magnitude of the average velocity is not called the average speed (as we will see in these next two examples).

> **Example 5** Assume that the runner in Example 3 completes the race in 1 minute and 18 seconds. Find her average speed and the magnitude of her average velocity.

Solution. *Average speed is total distance divided by elapsed time.* Since the length of the track is 500 m, the runner's average speed was (500 m)/(78 s) = 6.4 m/s. However, since her displacement was zero, her average velocity was zero also: $\bar{v} = \Delta x/\Delta t = (0 \text{ m})/(78 \text{ s}) = 0 \text{ m/s}$.

> **Example 6** Is it possible to move with constant speed but not constant velocity? Is it possible to move with constant velocity but not constant speed?

Solution. The answer to the first question is *yes*. For example, if you set your car's cruise control at 55 miles per hour but turn the steering wheel to follow a curved section of road, then the direction of your velocity changes (which means your velocity is not constant), even though your speed doesn't change.

The answer to the second question is *no*. Velocity means speed and direction; if the velocity is constant, then that means both speed and direction are constant. If speed were to change, then the velocity vector's magnitude would change.

ACCELERATION

When you step on the gas pedal in your car, the car's speed increases; step on the brake and the car's speed decreases. Turn the wheel, and the car's direction of motion changes. In all of these cases, the velocity changes. To describe this change in velocity, we need a new term: **acceleration.** In the same way that velocity measures the rate of change of an object's position, acceleration measures the rate of change of an object's velocity. An object's average acceleration is defined as follows:

$$\text{average acceleration} = \frac{\text{change in velocity}}{\text{time}}$$

$$\bar{a} = \frac{\Delta v}{\Delta t}$$

$$\text{units} = \text{meters/second}^2 = \text{m/s}^2$$

Note that an object can accelerate even if its speed doesn't change. (Again, it's a matter of not allowing the everyday usage of the word *accelerate* to interfere with its technical, physics usage.) This is because acceleration depends on Δv, and the velocity vector v changes if (1) speed changes, or (2) direction changes, or (3) both speed and direction change. For instance, a car traveling around a circular racetrack is constantly accelerating even if the car's *speed* is constant, because the direction of the car's velocity vector is constantly changing.

Remember: Velocity Is a Vector
Acceleration is a change in velocity meaning a change in magnitude or direction.

Example 7 A car is traveling in a straight line along a highway at a constant speed of 80 miles per hour for 10 seconds. Find its acceleration.

Solution. Since the car is traveling at a constant velocity, its acceleration is zero. If there's no change in velocity, then there's no acceleration.

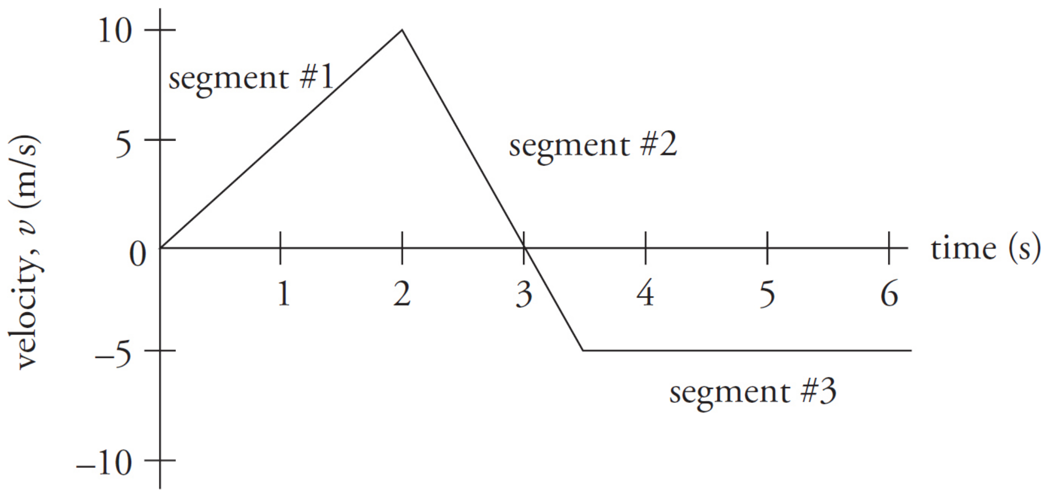
Example 8 A car is traveling along a straight highway at a speed of 20 m/s. The driver steps on the gas pedal and, 3 seconds later, the car's speed is 32 m/s. Find its average acceleration.

Solution. Assuming that the direction of the velocity doesn't change, it's simply a matter of dividing the change in velocity, $v_f - v_i$, 32 m/s – 20 m/s = 12 m/s, by the time interval during which the change occurred: $\overline{a} = \Delta v/\Delta t = (12 \text{ m/s})/(3 \text{ s}) = 4 \text{ m/s}^2$.

Example 9 Spotting a police car ahead, the driver of the car in Example 8 slows from 32 m/s to 20 m/s in 2 seconds. Find the car's average acceleration.

Solution. Dividing the change in velocity, 20 m/s – 32 m/s = –12 m/s, by the time interval during which the change occurred, 2 s, gives us $\overline{\mathbf{a}} = \Delta\mathbf{v}/\Delta t = (-12 \text{ m/s})/(2 \text{ s}) = -6 \text{ m/s}^2$. The negative sign here means that the direction of the acceleration is opposite the direction of the velocity, which describes slowing down.

Let's next consider an object moving along a straight axis in such a way that its velocity, v, as a function of time, t, is given by the following velocity-versus-time graph.



What does this graph tell us? It says that, at time t = 0, the object's velocity was v = 0. Over the first two seconds, its velocity increased steadily to 10 m/s. At time t = 2 s, the velocity then began to decrease (eventually becoming v = 0, at time t = 3 s). The velocity then became negative after t= 3 s, reaching v = −5 m/s at time t = 3.5 s. From t = 3.5 s on, the velocity remained a steady −5 m/s.

Segment #1: What can we ask about this motion? First, the fact that the velocity changed from t= 0 to t = 2 s tells us that the object accelerated. The acceleration during this time was

$$a = \frac{\Delta v}{\Delta t} = \frac{(10-0)\text{ m/s}}{(2-0)\text{ s}} = 5\text{ m/s}^2$$

Note, however, that the ratio that defines the acceleration, $\Delta v/\Delta t$, also defines the slope of the velocity-versus-time graph. Therefore,

The slope of a velocity-versus-time graph gives the acceleration.

Segment #2: What was the acceleration from time t = 2 s to time t = 3.5 s? The slope of the line segment joining the point (t, v) = (2 s, 10 m/s) to the point (t, v) = (3.5 s, −5 m/s) is

$$a = \frac{\Delta v}{\Delta t} = \frac{(-5-10)\text{ m/s}}{(3.5-2)\text{ s}} = -10\text{ m/s}^2$$

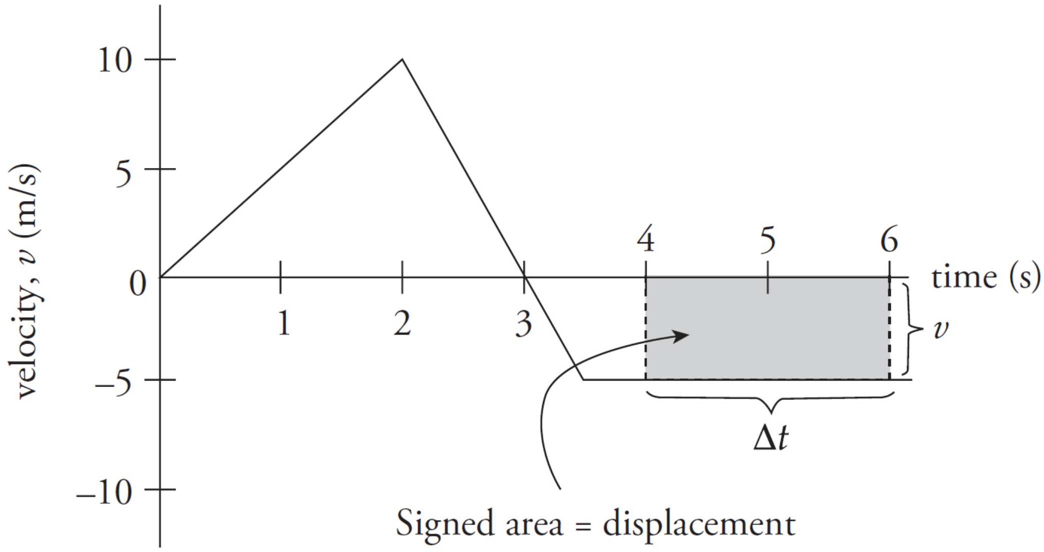
Between time t = 2 s to time t = 3.5 s, the object experienced a negative acceleration. Between time t = 2 s to time t = 3 s, the object's velocity was slowed down till it reached 0 m/s. Between time t = 3 s to time t = 3.5 s, the object began increasing its velocity in the opposite direction.

Segment #3: After time t = 3.5 s, the slope of the graph is zero, meaning the object experienced zero acceleration. This, however, does not mean the object did not move since its velocity was constant and in the negative direction.

When the slope of a velocity-versus-time graph is zero, the acceleration is zero.

Let's look at the velocity versus time graph at the bottom of this page again: how far did the object travel during a particular time interval? For example, let's figure out the displacement of the object from time t = 4 s to time t = 6 s. During this time interval, the velocity was a constant −5 m/s, so the displacement was $\Delta x = v\Delta t$ = (−5 m/s) (2 s) = −10 m.

Geometrically, we've determined the area between the graph and the horizontal axis. After all, the area of a rectangle is *base* × *height* and, for the shaded rectangle shown below, the *base* is Δt, and the *height* is v. So, *base* × *height* equals $\Delta t \times v$, which is displacement.

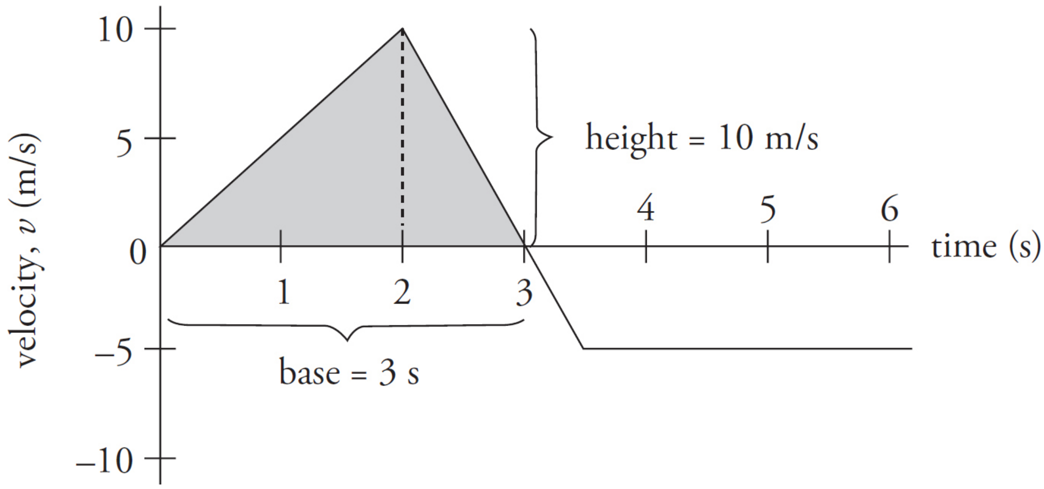


We say *signed area* because regions below the horizontal axis are negative quantities (since the object's velocity is negative, its displacement is negative). Thus, by counting areas above the horizontal axis as positive and areas below the horizontal axis as negative, we can make the following claim:

Given a velocity-versus-time graph, the area between the graph and the t-axis equals the object's displacement.

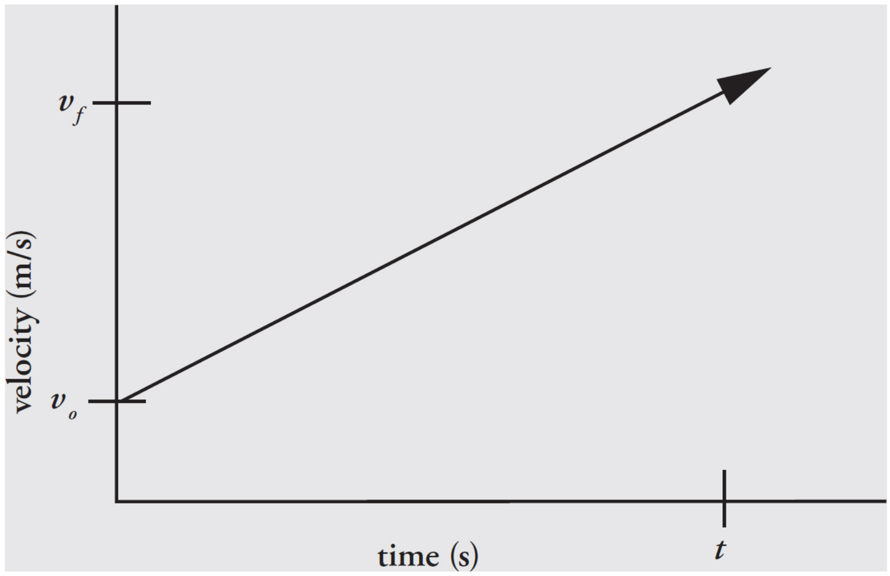
Not All Areas Are The Same!
Although the concept is strange, there are positive areas and negative areas. Think of area as a scalar quantity that can be negative and positive.

What is the object's displacement from time t = 0 to t = 3 s? Using the fact that displacement is the area bounded by the velocity graph, we figure out the area of the triangle shown below:

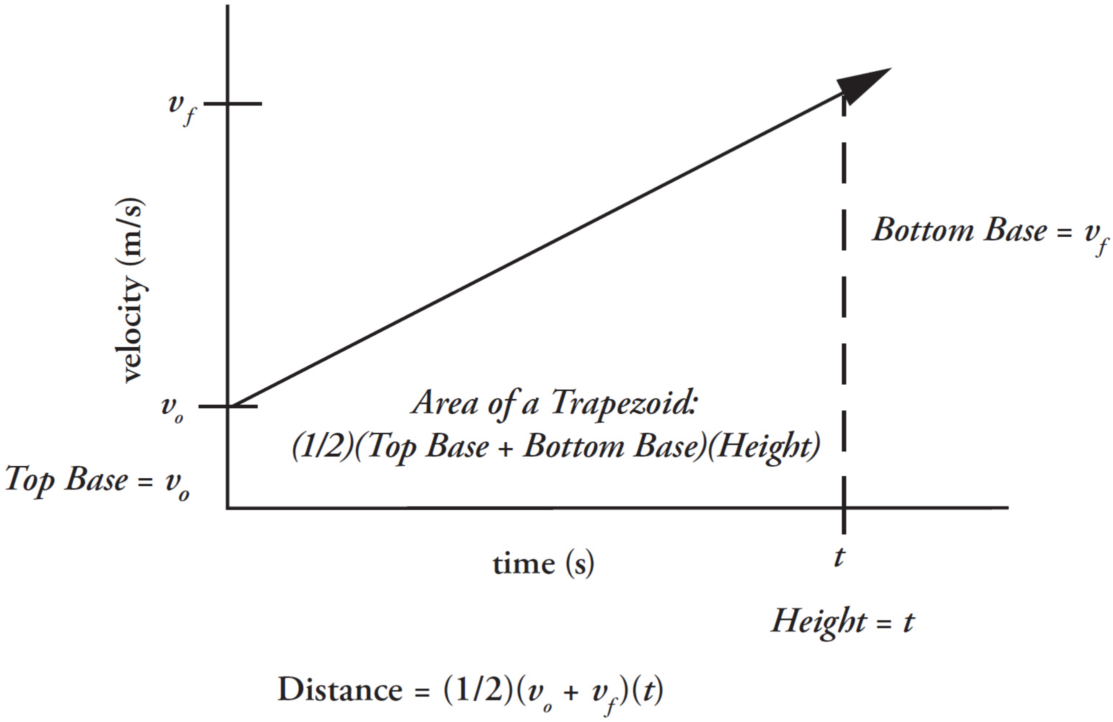


Since the area of a triangle is $\left(\frac{1}{2}\right)$ × base × height, we find that $\Delta x = \frac{1}{2}(3\text{ s})(10\text{ m/s}) = 15\text{ m}$.

Example 10 How far did the object travel from time 0 s to time t, given an initial velocity of v_0 and a final velocity of v_f in the graph below?



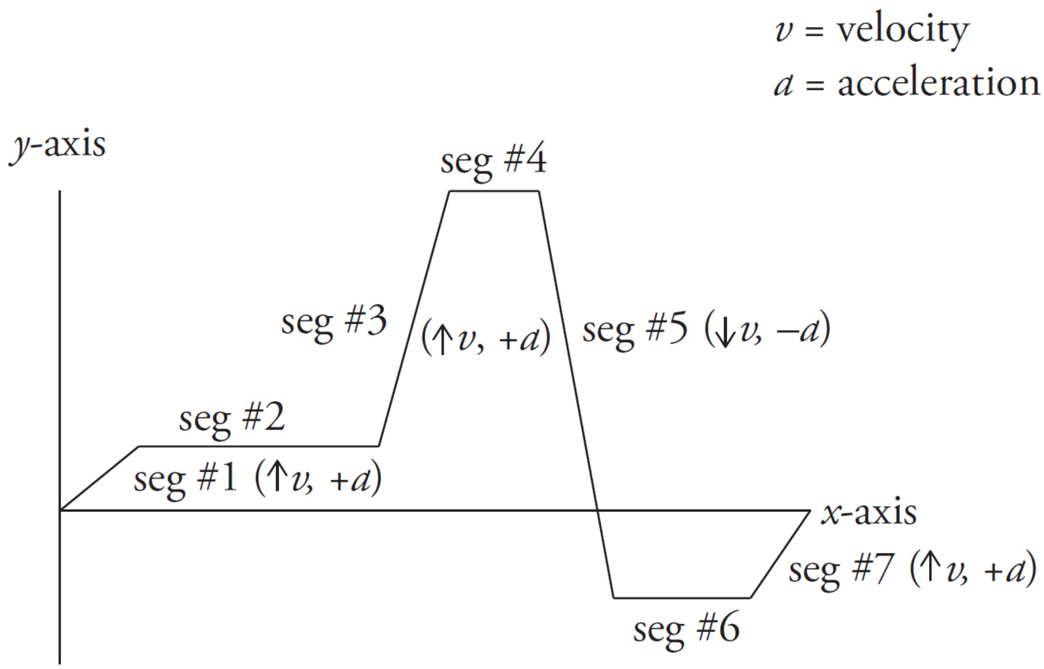
Solution.



Coming Soon

This is how the Big Five #1 equation is derived. More information about all that on the next page.

Let's consider the relationship between acceleration and velocity in a velocity-versus-time graph.



If we take an object's original direction of motion to be positive, then an increase in speed corresponds to positive acceleration. This is indicative of segment #1 and segment #3.

A decrease in velocity corresponds to negative acceleration (as indicated in top part of segment #5).

However, if an object's original direction is negative, then an increase in speed corresponds to negative acceleration, indicated by the bottom part of segment #5. Whatever is below the *x*-axis shows us that you are speeding up backward.

When the velocity and acceleration are in opposite directions, the object slows down, as is indicated in segment #7. Note that segments #2, #4, and #6 indicate no acceleration.

UNIFORMLY ACCELERATED MOTION AND THE BIG FIVE

The simplest type of motion to analyze is motion in which the acceleration is *constant,* also called **uniform accelerated motion.**

Another restriction that will make our analysis easier is to consider only motion that takes place along a straight line. In these cases, there are only two possible directions of motion. One is positive, and the opposite direction is negative. Most of the quantities we've been dealing with—displacement, velocity, and acceleration—are vectors, which means that they include both a magnitude and a direction. With straight-line motion, direction can be specified simply by attaching a + or – sign to the magnitude of the quantity. Therefore, although we will often abandon the use of bold letters to denote the vector quantities of displacement, velocity, and acceleration, the fact that these quantities include direction will still be indicated by a positive or negative sign.

> In the real world, truly uniform acceleration hardly occurs due to multiple factors. However, for the purposes of the AP Physics 1 Exam, everything is treated in ideal situations unless otherwise noted.

Let's review the quantities we've seen so far. The fundamental quantities are position (x), velocity (v), and acceleration (a). Acceleration is a change in velocity, from an initial velocity (v_i or v_0) to a final velocity (v_f or simply v—with no subscript). And, finally, the motion takes place during some elapsed time interval, Δt. Also, if we agree to start our clocks at time $t_i = 0$, then $\Delta t = t_f - t_i = t - 0 = t$, so we can just write t instead of Δt in the first four equations. This simplification in notation makes these equations a little easier to write down. Therefore, we have five kinematics quantities: Δx, v_0, v, a, and Δt.

When doing problems, writing down what is given and what you are solving for can help determine which Big Five equation to use.

These five quantities are related by a group of five equations that we call the *Big Five*. They work in cases where acceleration is uniform, which are the cases we're considering.

		Variable that's missing
Big Five #1:	$\Delta x = \frac{1}{2}(v_0 + v)t$	a
Big Five #2:	$v = v_0 + at$	x
Big Five #3:	$x = x_0 + v_0 t + \frac{1}{2}at^2$	v
Big Five #4:	$x = x_0 + vt - \frac{1}{2}at^2$	v_0
Big Five #5:	$v^2 = v_0^{\ 2} + 2a(x - x_0)$	t

Big Five #1 is the definition of velocity (this is the area under a velocity-versus-time graph, which will be covered a little further in this chapter). Big Five #2 is the definition of acceleration (this is the slope at any given moment of a velocity-versus-time graph).

Big Five #1 and #3 are simply the definitions of $\overline{v}$ and $\overline{a}$ written in different forms. The other Big Five equations can be derived from these two definitions.

Equations #1, #2, and #5 are the important three that can be used to solve any problem (equations #3 and 4 are derivations). However, it is advisable to memorize all five equations to speed up problem-solving during the test.

Example 11 An object with an initial velocity of 4 m/s moves along a straight axis under constant acceleration. Three seconds later, its velocity is 14 m/s. How far did it travel during this time?

Solution. We're given v_0, Δt, and v, and we're asked for x. So a is missing; it isn't given and it isn't asked for, so we use Big Five #1:

$$x = \overline{v}t = \frac{1}{2}(v_0 + v)t = \frac{1}{2}(4 \text{ m/s} + 14 \text{ m/s})(3 \text{ s}) = 27 \text{ m}$$

Structured Example Method to Solve Problems

Given:

v(initial) = 4 m/s
v(final) = 14 m/s

$t = 3$ s

Missing:

acceleration (a)

Unknown (What we are solving for):

distance (x)

Equation:

Big Five #1

It's okay to leave off the units in the middle of the calculation as long as you remember to include them in your final answer. **Leaving units off your final answer will cost you points on the AP Exam.**

Example 12 A car that's initially traveling at 10 m/s accelerates uniformly for 4 seconds at a rate of 2 m/s², in a straight line. How far does the car travel during this time?

Solution. We're given v_0, t, and a, and we're asked for x. So, v is missing; it isn't given, and it isn't asked for. Therefore, we use Big Five #3:

$$x = x_0 + v_0\Delta t + \frac{1}{2}a(\Delta t)^2 = (10 \text{ m/s})(4 \text{ s}) + \frac{1}{2}(2 \text{ m/s}^2)(4 \text{ s})^2 = 56 \text{ m}$$

Example 13 A rock is dropped off a cliff that's 80 m high. If it strikes the ground with an impact velocity of 40 m/s, what acceleration did it experience during its descent?

Solution. If something is dropped, then that means it has no initial velocity: $v_0 = 0$. So, we're given v_0, Δx, and v, and we're asked for a. Since t is missing, we use Big Five #5:

$$v^2 = v_0^2 + 2a(x - x_0) \Rightarrow v^2 = 2a(x - x_0) \quad (\text{since } v_0 = 0)$$

$$a = \frac{v^2}{2(x - x_0)} = \frac{(40 \text{ m/s})^2}{2(80 \text{ m})} = 10 \text{ m/s}^2$$

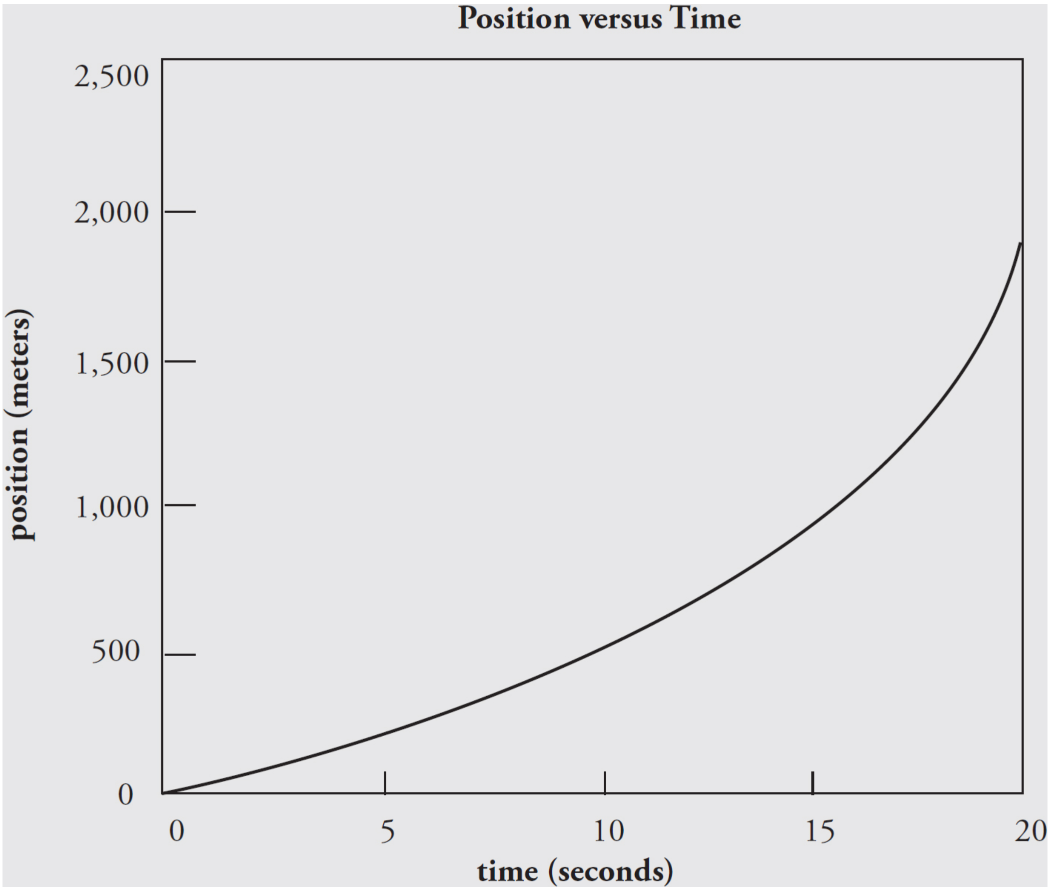
Note that since a has the same sign as $(x - x_0)$, the acceleration vector points in the same direction as the displacement vector. This makes sense here, since the object moves downward, and the acceleration it experiences is due to gravity, which also points downward.

> **Initial Velocity?**
> Some problems do not give initial velocity in numerical terms. If you see the statement "starting from rest" or "dropped," we can assume our initial velocity is zero.

ADDITIONAL KINEMATIC GRAPHICAL ASPECTS

Not all graphs have nice straight lines as shown so far. Straight line segments represent constant slopes and therefore constant velocities, or accelerations, depending on the type of graph. What happens as an object changes its velocity in a position-versus-time graph? The "lines" become "curves." Let's look at a typical question that might be asked about such a curve.

Example 14



a) What is the average velocity from 0 to 10 seconds?

b) What is the average velocity from 10 to 20 seconds?

c) What is the average velocity from 0 to 20 seconds?

Solution. This is familiar territory. To find the average velocity, use

a) $v_{avg} = \dfrac{\Delta x}{\Delta t} \Rightarrow \dfrac{(500-0)}{(10-0)} \Rightarrow 50 \text{ m/s}$

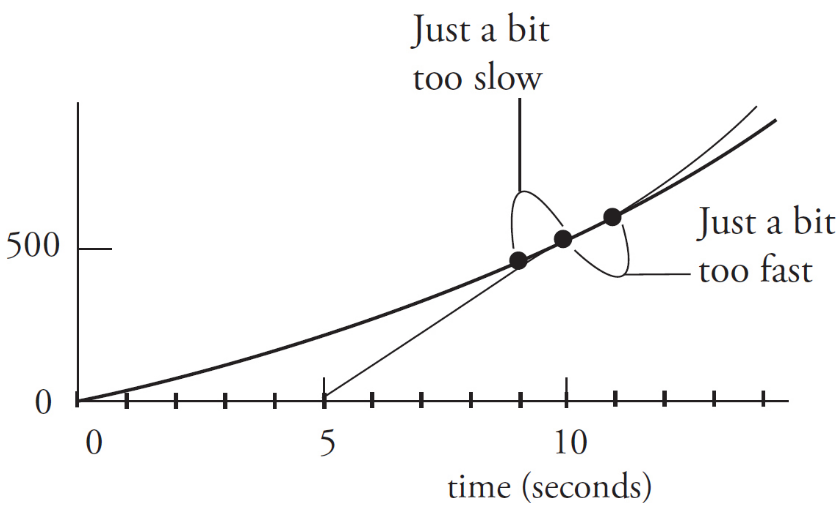
b) $v_{avg} = \dfrac{\Delta x}{\Delta t} \Rightarrow \dfrac{(2{,}000-500)}{(20-10)} \Rightarrow 150 \text{ m/s}$

c) $v_{avg} = \frac{\Delta x}{\Delta t} \Rightarrow \frac{(2{,}000-0)}{(20-0)} \Rightarrow 100 \text{ m/s}$

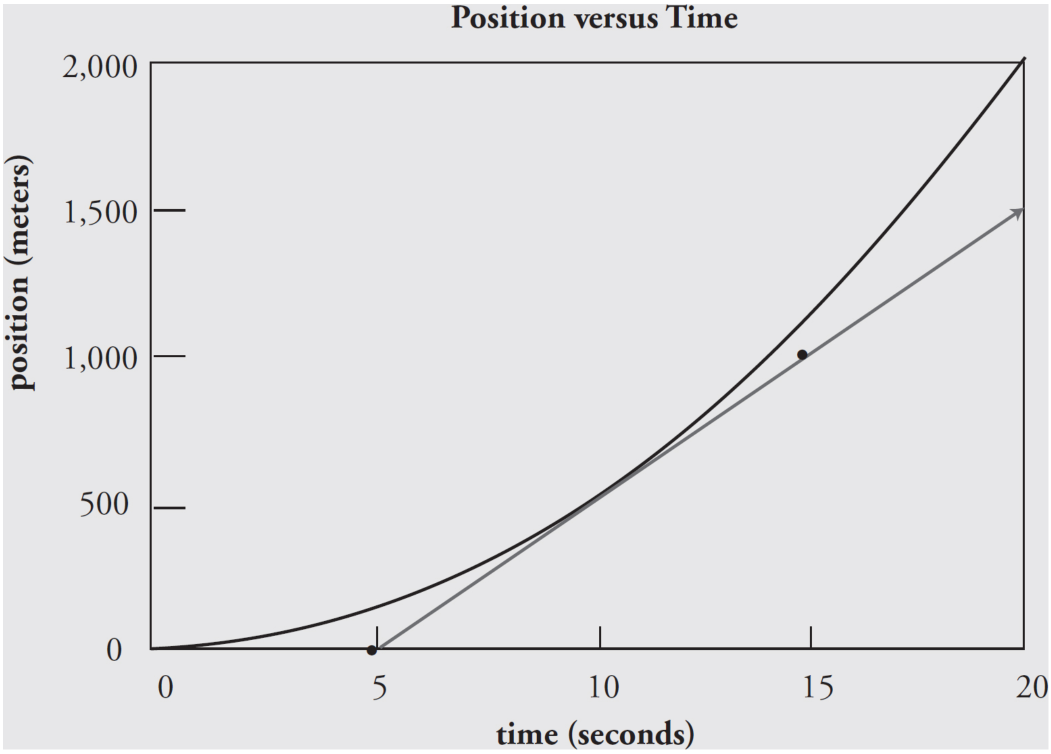
The instantaneous velocity is the velocity at a given moment in time. When you drive in a car and look down at the speedometer, you see the magnitude of your instantaneous velocity at that time.

What Is a Tangent?
A line that touches a curved line at only one point.

To find the instantaneous velocity at 10 seconds, we will need to approximate. The velocity from 9–10 seconds is close to the velocity at 10 seconds, but it is still a bit too slow. The velocity from 10–11 seconds is also close to the velocity at 10 seconds, but it is still a bit too fast. You can find the middle ground between these two ideas, or the slope of the line that connects the point before and the point after 10 seconds. This is very close to the instantaneous velocity. A true tangent line touches the curve at only one point, but this line is close enough for our purposes.



Example 15 Using the graph below, what is the instantaneous velocity at 10 seconds?



Solution. Draw a tangent line. Find the slope of the tangent by picking any two points on the tangent line. It usually helps if the points are kind of far apart and it also helps if points can be found at "easy spots" such as (0, 5) and (15, 1,000) and not (6.27, 113) and (14.7, 983):

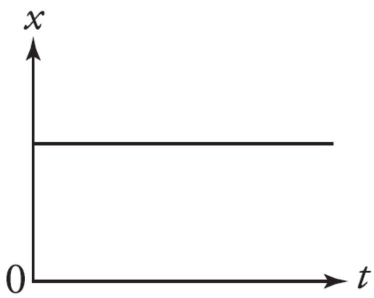
$$v_{ins} = v_{\tan} = \frac{\Delta x}{\Delta t} \Rightarrow \frac{(1{,}000-0)}{(15-5)} \Rightarrow 100 \text{ m/s}$$

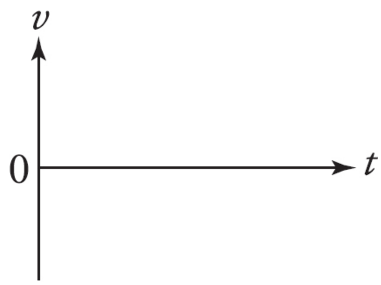
QUALITATIVE GRAPHING

Beyond all the math, you are at a clear advantage when you start to recognize that position-versus-time and velocity-versus-time graphs have a few basic shapes, and that all the graphs you will see will be some form of these basic shapes. Having a feel for these building blocks will go a long way toward understanding kinematics graphs in physics.

> **No Calculus**
> You may have seen the relationships of displacement, velocity, and acceleration described using the words "derivative" or "integral." That's calculus stuff that you'll learn one day (or may have learned already), but you don't need any of it for this test.

Either of the following two graphs represents something that is not moving.

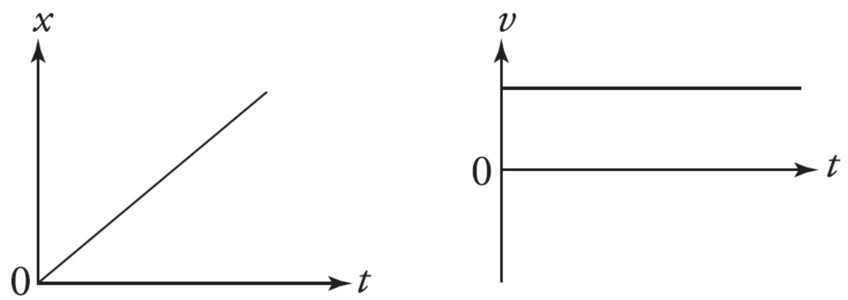




- no change in position

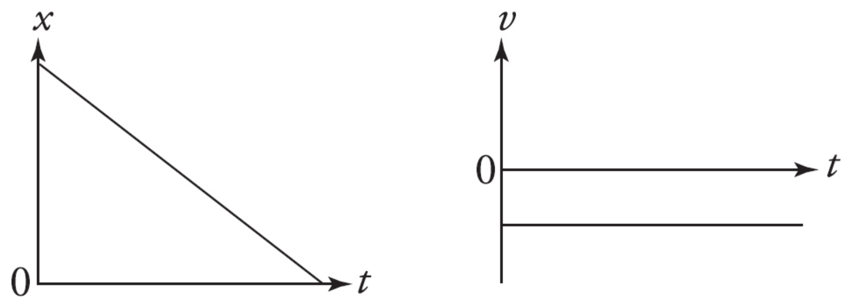
- zero velocity
- zero acceleration

Either of the following two graphs represents an object moving at a constant velocity in the positive direction.



- positive change in position
- constant velocity
- zero acceleration

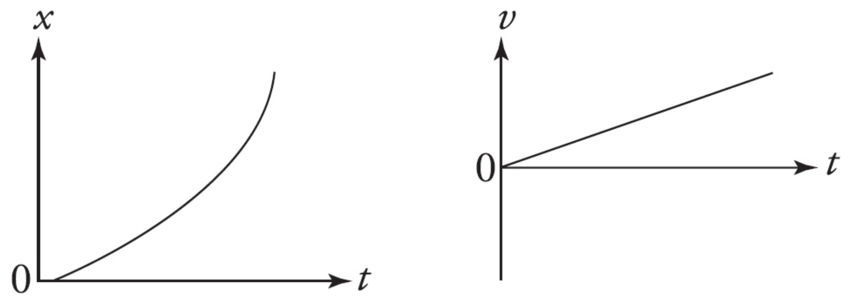
Either of the following two graphs represents an object moving at a constant velocity in the negative direction.



- negative change in position
- constant velocity
- zero acceleration

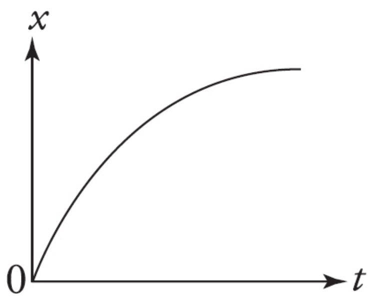
Know These Graphs
Familiarize yourself with these graphs so that you can quickly look at any graph and have a sense of what's going on as far as change in position, velocity, and acceleration immediately.

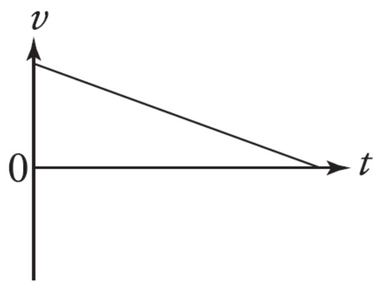
Either of the following two graphs represents an object speeding up in the positive direction.



- positive change in position
- increasing velocity
- positive acceleration

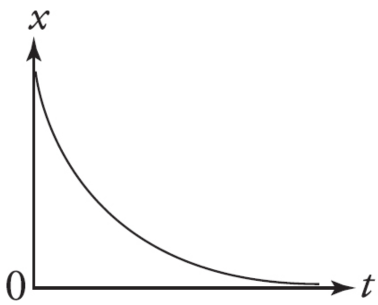
Either of the following two graphs represents an object slowing down in the positive direction.

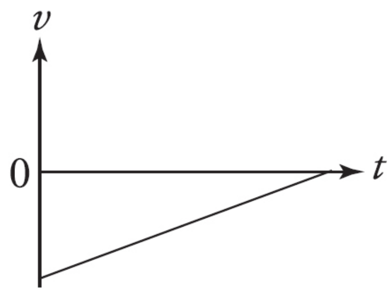




- positive change in position
- decreasing positive velocity
- negative acceleration

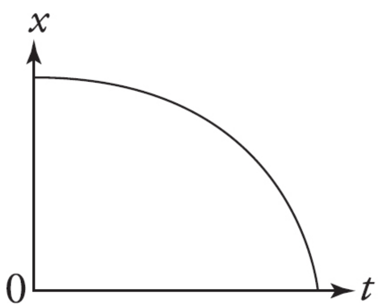
Either of the following two graphs represents an object slowing down in the negative direction.

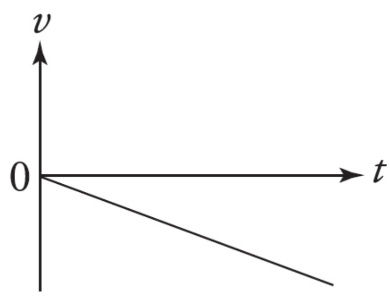




- negative change in position
- decreasing negative velocity
- positive acceleration

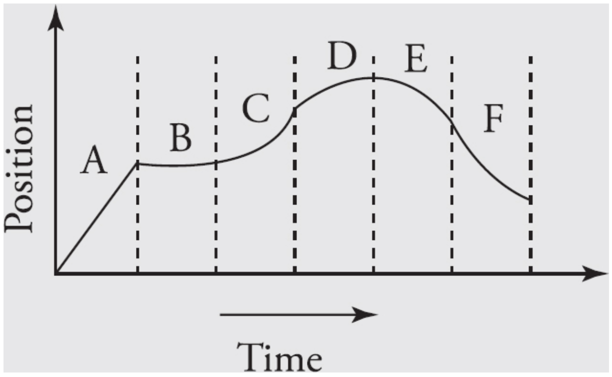
Either of the following two graphs represents an object speeding up in the negative direction.





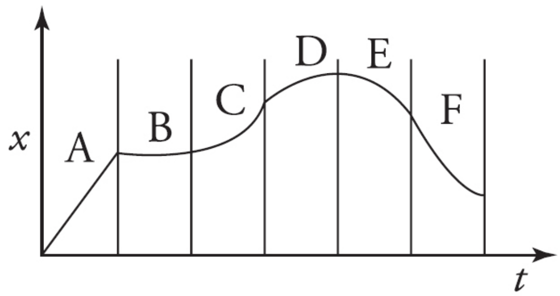
- negative change in position
- increasing negative velocity
- negative acceleration

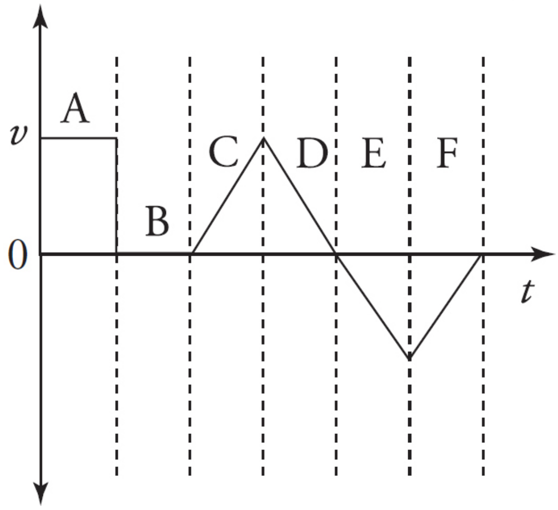
Example 16 Below is a position-versus-time graph. Describe in words the motion of the object and sketch the corresponding velocity-versus-time graph.



Solution. Part A is a constant speed moving away from the origin, part B is at rest, part C is speeding up moving away from the origin, part D is slowing down still moving away from the origin, part E is speeding up moving back toward the origin, and part F is slowing down moving back toward the origin.

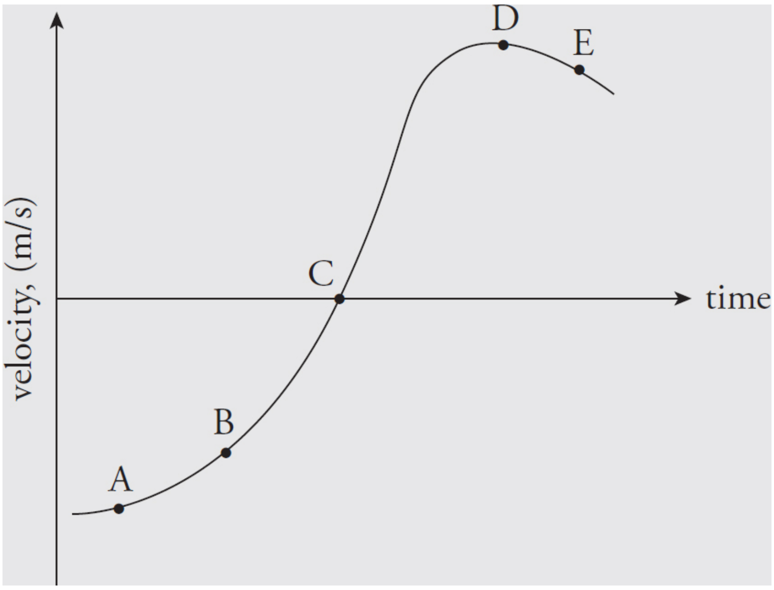
Curves in a Position-Versus-Time Graph
Curving up represents a positive acceleration. Curving down represents a negative acceleration.





The area under an acceleration-versus-time (also referred to as an a-versus-t) graph gives the change in velocity.

Example 17 The velocity of an object as a function of time is given by the following graph:



At which point (A, B, C, D, or E) is the magnitude of the acceleration the greatest?

One Slope to Rule Them All

The greater the slope at a point of a velocity-versus-time graph, the greater the acceleration is.

The greater the slope at a point of a position-versus-time graph, the greater the velocity is.

Solution. The acceleration is the slope of the velocity-versus-time graph. Although this graph is not composed of straight lines, the concept of slope still applies; at each point, the slope of the curve is the slope of the tangent line to the curve. The slope is essentially zero at Points A and D (where the curve is flat), small and positive at B, and small and negative at E. The slope at Point C is large and positive, so this is where the object's acceleration is the greatest.

FREE FALL

The simplest real-life example of motion under pretty constant acceleration is the motion of objects in Earth's gravitational field, near the surface of the Earth and ignoring any effects due to the air (mainly air resistance). With these effects ignored, an object can fall *freely*. That is, it can fall experiencing only acceleration due to gravity. Near the surface of the Earth, the gravitational acceleration has a constant magnitude of about 9.8 m/s^2; this quantity is denoted g (for *g*ravitational acceleration). On the AP Physics 1 Exam, you may use g = 10 m/s^2 as a simple approximation to g = 9.8 m/s^2. Even though you can use the exact value on your calculator, this approximation helps to save time, so in this book, we will always use g = 10 m/s^2. And, of course, the gravitational acceleration vector, **g,** points *downward*.

Since gravity points downward, for the sake of keeping things consistent throughout the rest of this chapter, we will set gravity as g = –10 m/s^2.

Example 18 A rock is dropped from a cliff 80 m above the ground. How long does it take to reach the ground?

Solution. We are given v_0 and asked for t. Unless you specifically see words to the contrary (such as, "you are on the moon where the acceleration due to gravity is…"), assume you are also given $a = -10\ \text{m/s}^2$. Because v is missing, and it isn't asked for, we can use Big Five equation #3.

$$y = y_0 + v_0 t + \frac{1}{2}at^2 \Rightarrow y = \frac{1}{2}at^2$$

If we set the origin $y_0 = 0$ at the base of the cliff and $v_0 = 0$, we get

$$t = \sqrt{\frac{2y}{a}}$$

$$t = \sqrt{\frac{2(-80\ \text{m})}{(-10\ \text{m/s}^2)}} = 4\ \text{s}$$

Note: The negative in front of the 80 is inserted because the rock fell in the down direction.

A Note About Gravity
We don't always have to set gravity as negative. You are allowed to choose any coordinate system you wish as long as you match the positive or negative with gravity correctly.

Example 19 A baseball is thrown straight upward with an initial speed of 20 m/s. How high will it go?

The time it takes for an object to be thrown straight up is the same as the time it takes for an object to fall down from the peak back into your hand.

Solution. We are given v_0, $a = -10\ \text{m/s}^2$ is implied, and we are asked for y. Now, neither t nor $v$is expressly given; however, we know the vertical velocity at the top is 0 (otherwise the baseball would still rise). Consequently, we use Big Five equation #5.

$$v^2 = v_0^2 + 2a(y - y_0) \Rightarrow -2ay = v_0^2$$

We set $y_0 = 0$ and we know that $v = 0$, so that leaves us with

$$y = -\frac{v_0^2}{2a}$$

$$y = -\frac{(20\ \text{m/s})^2}{2\left(-10\ \text{m/s}^2\right)} = 20\ \text{m}$$

Example 20 One second after being thrown straight down, an object is falling with a speed of 20 m/s. How fast will it be falling 2 seconds later?

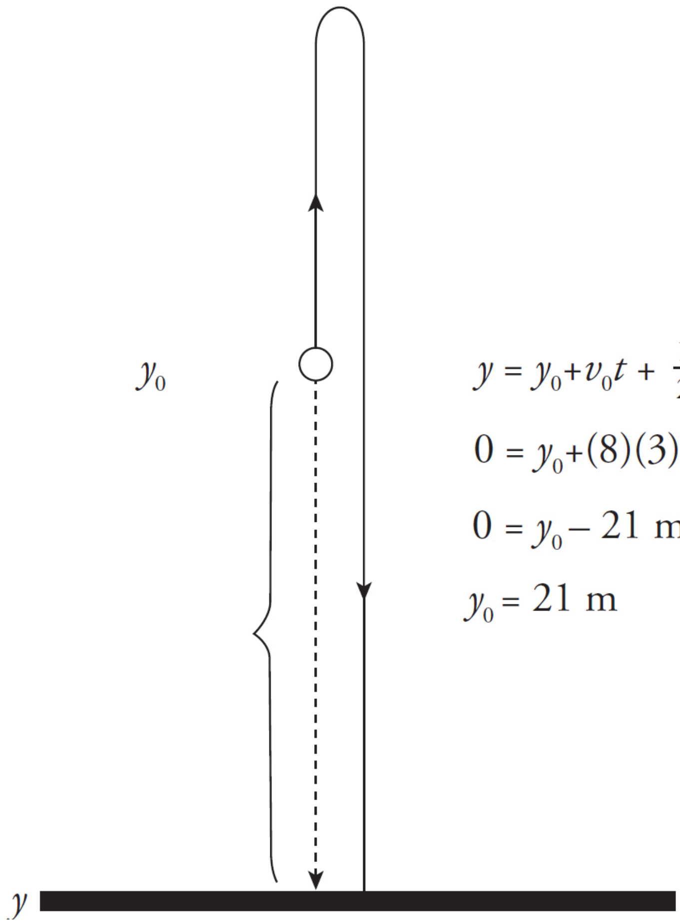
Solution. We're given v_0, a, and t and asked for v. Since x is missing, we use Big Five #2:

$$v = v_0 + at = (-20 \text{ m/s}) + (-10 \text{ m/s}^2)(2 \text{ s}) = -40 \text{ m/s}$$

The negative sign in front of the 40 simply indicates that the object is traveling in the down direction.

> **Example 21** If an object is thrown straight upward with an initial speed of 8 m/s and takes 3 seconds to strike the ground, from what height was the object thrown?

Solution. We're given a, v_0, and t and we need to find y_0. Because v is missing, we use Big Five #3:



$$y = y_0 + v_0 t + \frac{1}{2}at^2$$

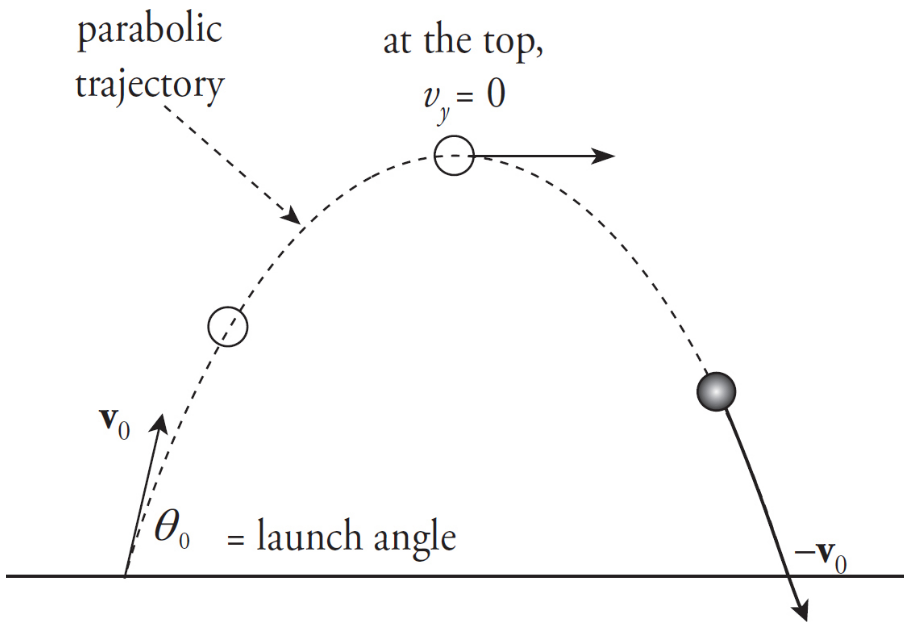
$$0 = y_0 + (8)(3) + \frac{1}{2}(-10)(3^2)$$

$$0 = y_0 - 21 \text{ m}$$

$$y_0 = 21 \text{ m}$$

PROJECTILE MOTION

In general, an object that moves near the surface of the Earth will not follow a straight-line path (for example, a baseball hit by a bat, a golf ball struck by a club, or a tennis ball hit from the baseline). If we launch an object at an angle other than straight upward and consider only the effect of acceleration due to gravity, then the object will travel along a parabolic trajectory.



To simplify the analysis of parabolic motion, we analyze the horizontal and vertical motions separately, using the Big Five. This is the key to doing **projectile motion** problems.

A Note About Vectors
Vectors that are perpendicular to each other do not affect each other's magnitudes, only their directions.

Horizontal motion:

$\Delta x = v_{ox}t$

$v_x = v_{ox}$ (constant!)

$a_x = 0$

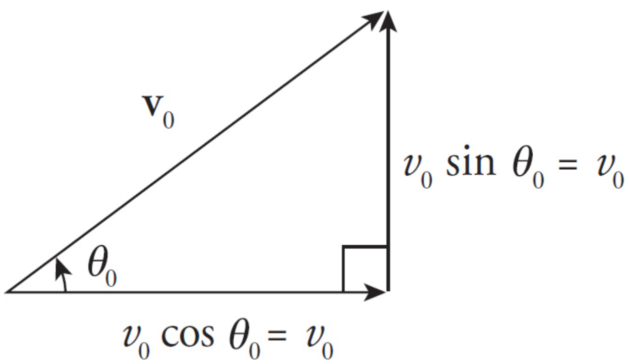
Vertical motion:

$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$

$v_y = v_{0y} - gt$

$a_y = -g = -10 \text{ m/s}^2$

The quantity v_{ox}, which is the horizontal (or x) component of the initial velocity, is equal to $v_o \cos \theta_o$, where θ_o is the **launch angle,** the angle that the initial velocity vector, $\mathbf{v}_o$, makes with the horizontal. Similarly, the quantity v_{oy}, the vertical (or y) component of the initial velocity, is equal to $v_o \sin \theta_o$.



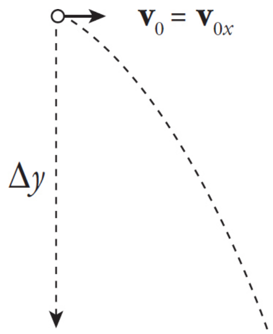
Note that time is the one value that is always the same for both the horizontal and vertical motion. If you're lost on a projectile motion question, solving for time is usually a good start.

Example 22 An object is thrown horizontally with an initial speed of 10 m/s. It hits the ground 4 seconds later. How far did it drop in 4 seconds?

Solution. The first step is to decide whether this is a *horizontal* question or a *vertical* question, since you must consider these motions separately. The question *How far did it drop?* is a *vertical*question, so the set of equations we will consider are those listed on the previous page under *vertical motion*. Next, *How far...?* implies that we will use the first of the vertical-motion equations, the one that gives vertical displacement, Δy.

REMEMBER THIS!
Horizontal velocity in standard parabolic motion is always constant.

Now, since the object is thrown horizontally, there is no vertical component to its initial velocity vector $\mathbf{v}_0$; that is, $v_{0y} = 0$. Therefore,



$$\Delta y = v_{0y}t - \frac{1}{2}gt^2 \rightarrow \Delta y = -\frac{1}{2}gt^2 \quad (\text{because } v_{0y} = 0)$$
$$= -\frac{1}{2}(10)(4^2)$$
$$= -80 \text{ m}$$

The fact that Δy is negative means that the displacement is *down*. Also, notice that the information given about v_{0x} is irrelevant to the question.

Example 23 From a height of 100 m, a ball is thrown horizontally with an initial speed of 15 m/s. How far does it travel horizontally in the first 2 seconds?

Solution. The question, *How far does it travel horizontally...?*, immediately tells us that we should use the first of the horizontal-motion equations:

$$\Delta x = v_{0x}t = (15 \text{ m/s})(2 \text{ s}) = 30 \text{ m}$$

The information that the initial vertical position is 100 m above the ground is irrelevant (except for the fact that it's high enough that the ball doesn't strike the ground before the two seconds have elapsed).

Example 24 A projectile is traveling in a parabolic path for a total of 6 seconds. How does its horizontal velocity 1 s after launch compare to its horizontal velocity 4 s after launch?

Solution. The only acceleration experienced by the projectile is due to gravity, which is purely vertical, so that there is no horizontal acceleration. If there's no horizontal acceleration, then the horizontal velocity cannot change during flight, and the projectile's horizontal velocity 1 s after it's launched is the same as its horizontal velocity 3 s later.

For the vertical component of parabolic motion, you can treat it as throwing an object straight up and it falling back down. Just remember that the object traveling to its peak is only a part of its time of travel. It still needs to fall.

Example 25 An object is projected upward with a 30° launch angle and an initial speed of 40 m/s. How long will it take for the object to reach the top of its trajectory? How high is this?

Solution. When the projectile reaches the top of its trajectory, its velocity vector is momentarily horizontal; that is, v_y = 0. Using the vertical-motion equation for v_y, we can set it equal to 0 and solve for *t*:

$$v_y \overset{\text{set}}{=} 0 \Rightarrow v_{0y} - gt = 0$$

$$t = \frac{v_{0y}}{g} = \frac{v_0 \sin \theta_0}{g} = \frac{(40 \text{ m/s}) \sin 30^\circ}{10 \text{ m/s}^2} = 2 \text{ s}$$

At this time, the projectile's vertical displacement is

$$\begin{aligned}\Delta y &= v_{0y} t - \frac{1}{2}(g)t^2 = (v_0 \sin\theta_0)t - \frac{1}{2}(g)t^2 \\ &= \left[(40 \text{ m/s}) \sin 30^\circ\right](2 \text{ s}) - \frac{1}{2}(10 \text{ m/s}^2)(2 \text{ s})^2 \\ &= 20 \text{ m}\end{aligned}$$

Example 26 An object is projected upward with a 30° launch angle from the ground and an initial speed of 60 m/s. For how many seconds will it be in the air? How far will it travel horizontally? Assume it returns to its original height.

Solution. The total time the object spends in the air is equal to twice the time required to reach the top of the trajectory (because the parabola is symmetrical). So, as we did in the previous example, we find the time required to reach the top by setting v_y equal to 0, and now double that amount of time:

$$v_y \overset{\text{set}}{=} 0 \Rightarrow v_{0y} - gt = 0$$

$$t = \frac{v_{0y}}{g} = \frac{v_0 \sin\theta_0}{g} = \frac{(60 \text{ m/s}) \sin 30^\circ}{10 \text{ m/s}^2} = 3 \text{ s}$$

Because the motion in this example is parabolic, $v_f = -v_0$. If you use that value for v_f in the equations here, you can solve for the entire flight time (making it unnecessary to multiply by 2 at the end). Both methods give the correct solution, so use whichever you find easier.

Therefore, the *total* flight time (that is, up and down) is $t_t = 2t = 2 \times (3\text{ s}) = 6\text{ s}$.

Now, using the first horizontal-motion equation, we can calculate the horizontal displacement after 6 seconds:

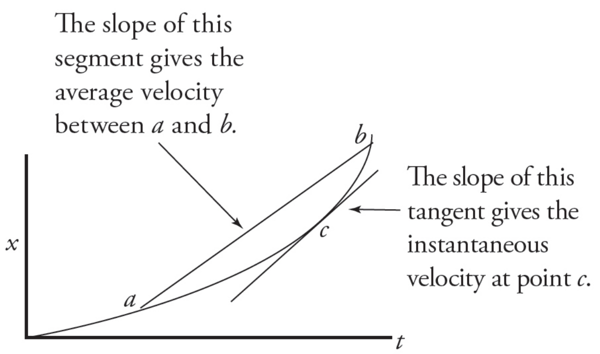
$$\Delta x = v_{0x} t_t = \left(v_0 \cos\theta_0\right) t_t = \left[\left(60\text{ m/s}\right)\cos 30^\circ\right]\left(6\text{ s}\right) = 312\text{ m}$$

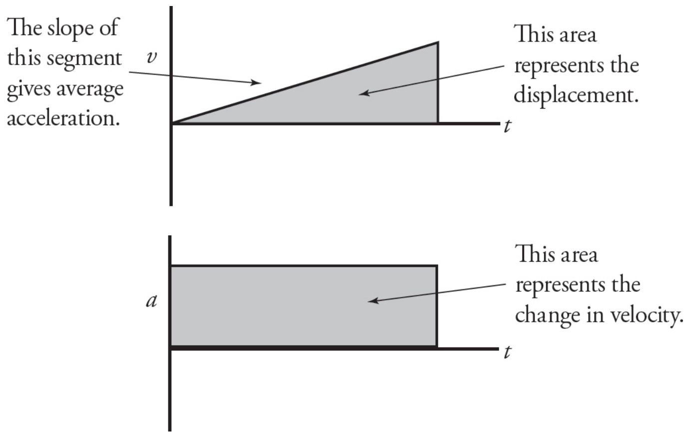
By the way, assuming it lands back at its original height, the full horizontal displacement of a projectile is called the projectile's **range.**

Summary

Graphs are very useful tools to help visualize the motion of an object. They can also help you solve problems once you learn how to translate from one graph to another. As you work with graphs, keep the following things in mind:

- Always look at a graph's axes first. This sounds obvious, but one of the most common mistakes students make is looking at a velocity-versus-time graph, thinking about it as if it were a position-versus-time graph.
- Don't ever assume one box is one unit. Look at the numbers on the axes.
- Lining up position-versus-time graphs *directly above* velocity-versus-time graphs and *directly above* acceleration-versus-time graphs is a must. This way you can match up key points from one graph to the next.
- The slope of an x versus t graph gives velocity. The slope of a v versus t graph gives acceleration.
- The area under an acceleration-versus-time graph gives the change in velocity. The area under a velocity-versus-time graph gives the displacement.





- The motion of an object in one dimension can be described using the Big Five equations. Look for what is given, determine what you're looking for, and use the equation that has those variables in them. Remember that sometimes there is hidden (assumed) information in the problem, such as $a = -10\ \text{m/s}^2$.

Name of Equation	Equation	Missing Variable
Big Five #1:	$\Delta x = \frac{1}{2}(v_0 + v)t$	a
Big Five #2:	$v = v_0 + at$	x
Big Five #3:	$x = x_0 + v_0 t + \frac{1}{2}at^2$	v
Big Five #4:	$x = x_0 + vt - \frac{1}{2}at^2$	v_0
Big Five #5:	$v^2 = v_0^2 + 2a(x - x_0)$	t

- For projectiles, it is important to separate the horizontal and vertical components.

Horizontal Motion:

$x = v_x t$

Vertical Motion:

$y = y_0 + v_0 t + \frac{1}{2}gt^2$

$v_x = v_{0x} = \text{constant}$

$a_x = 0$

$v_y = v_{0y} + gt$

$a = g = -10 \text{ m/s}^2$

- At any given moment, the relationship between v, v_x, and v_y is given by

$$v_x = v\cos\theta \qquad v_y = v\sin\theta$$

$$v^2 = v_x{}^2 + v_y{}^2 \quad \text{and} \quad \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$