## AP Physics - Thermodynamics Wrap-up

Here are your basic equations for thermodynamics. There's a bunch of them.
$T_{C}=\frac{5}{9}\left(T_{F}-32^{0}\right)$
This equation converts temperature from Fahrenheit to Celsius.
$H=\frac{Q}{t}=\frac{k A \Delta T}{L}$
This is the rate of heat transfer for conduction equation. The rate of heat flow is directly dependent on the surface area in contact and the temperature difference. It is inversely proportional to the thickness of the material.

$$
p V=n R T
$$

The good old ideal gas law.
$U=\frac{3}{2} k_{b} T$
This equation will calculate the average internal energy of a gas particle for a given temperature. $k_{B}$ is Boltzmann's constant. $k_{b}=1.38 \times 10^{-23} \frac{J}{K}$
$v_{r m s}=\sqrt{\frac{3 R T}{M}}=\sqrt{\frac{3 k_{b} T}{m}}$
This equation calculates the average velocity (symbol $v_{r m s}$ ) which is actually referred to as the root-means-square velocity for a gas particle. $R$ is the universal gas constant $(\mathrm{R}=8.31$ $\mathrm{J} / \mathrm{mol} \cdot \mathrm{K}$ ), $M$ is the molecular mass (mass on one mole of an element), and m is the mass of a molecule (this would be the mass of a single molecule), and the rest of the stuff you know.
$\Delta l=\alpha l_{0} \Delta T$
This is the equation for the change in length of an object as a function of temperature. The $\alpha$ is the coefficient of linear expansion, $l_{0}$ is the initial length of the object, $\Delta l$ is the change in length from the expansion, and $\Delta T$ is the temperature difference.
$\Delta A=2 \alpha A_{0} \Delta T$
This is the equation for the change in area of an object as a function of temperature. The $\alpha$ is the coefficient of linear expansion, $\mathrm{A}_{0}$ is the initial area of the object, $\Delta \mathrm{A}$ is the change in area from the expansion, and $\Delta T$ is the temperature difference.
$\Delta V=\beta V_{0} \Delta T$
This is the equation for the change in volume of an object as a function of temperature. The $\beta$ is the coefficient of volume expansion, $\mathrm{V}_{0}$ is the initial volume of the object, $\Delta \mathrm{A}$ is the change in volume from the expansion, and $\Delta T$ is the temperature difference.
$W=-p \Delta V$
This is the work done by a system that expands at a constant pressure - your basic piston/cylinder heat engine deal.
$\Delta U=Q+W$
This is the first law of thermodynamics. The change in the internal energy of a system is equal to the heat added to the system plus the work done on the system.
$e=\left|\frac{W}{Q_{H}}\right|$
The general equation for the efficiency of a heat engine. Use this to find the efficiency of an engine - $\boldsymbol{W}$ is the work done by the engine and $\mathrm{Q}_{\mathrm{H}}$ is the upper heat reservoir (which represents the heat added to the system).
$e_{c}=\frac{T_{H}-T_{C}}{T_{H}}$
This is the ideal efficiency for a heat engine. This is the maximum possible value the engine can have. The efficiency is a function of the upper and lower operating temperature. It is important for you to realize that no real heat engine can actually achieve this efficiency.

Here's what you have to be doing with all this thermo stuff.
A. Temperature and Heat

1. You should understand the "mechanical equivalent of heat" so you can calculate how much a substance will be heated by the performance of a specified quantity of mechanical work.

This just means that you should be able to convert from calories to Joules and Joules to calories. It also involves the use of the law of conservation of energy. Mechanical work done on a thermodynamic system will increase its thermal energy etc... 1 cal = 4.186 J
2. You should understand heat transfer and thermal expansion so you can:
a. Calculate how the flow of heat through a slab of material is affected by changes in the thickness or area of the slab, or the temperature difference between the two faces of the slab.

The rate of heat transfer is given by the following equation: $H=\frac{Q}{t}=\frac{k A \Delta T}{L}$
b. Analyze what happens to the size and shape of a body when it is heated.

You have to say if it gets bigger or smaller, and precisely by what amount this would happen. You basically use the kinetic theory of matter. You could also use the equation for linear expansion, since objects expand in all directions when heated. $\quad \Delta l=a l_{0} \Delta T$ Basically a body will expand in all directions when heated. It does this because the particles that make it up move with greater energy, taking up more space.
c. Analyze qualitatively the effects of conduction, radiation, and convection in thermal processes.
"Qualitatively" means that you just have to conceptually understand each of these transfer types.
B. Kinetic Theory and Thermodynamics

1. Ideal Gases
a. You should understand the kinetic theory model of an ideal gas so you can:
(1) State the assumptions of the model.

Here they are:

- The number of particles in a system is enormous and the separation between the particles is huge.
- All the particles move randomly.
- The particles have perfectly elastic collisions with each other and other atoms.
- There are no forces of attraction between the particles of a gas.
- All the particles are identical.
(2) State the connection between temperature and mean translational kinetic energy, and apply it to determine mean speed of gas molecules as a function of their mass and the temperature of the gas.
The mean translational kinetic energy of a particle is given by: $K_{\text {avg }}=\frac{3}{2} k_{B} T$

The mean speed of a gas molecule is given by:

$$
v_{r m s}=\sqrt{\frac{3 R T}{M}}=\sqrt{\frac{3 k_{B} T}{\mu}}
$$

You had a chance to do many of these problems as part of your beloved homework. Previous units showed you how to do them; also you will be provided with the equations.
(3) State the relationship among Avogadro's number, Boltzmann's constant, and the gas constant R, and express the energy of a mole of a monatomic ideal gas as a function of its temperature.

Boltzmann's constant only shows up in two of our equations; $v_{r m s}=\sqrt{\frac{3 R T}{M}}=\sqrt{\frac{3 k_{B} T}{\mu}}$ and $K_{a v g}=\frac{3}{2} k_{B} T$

If you look at the first one, you see:
$\sqrt{\frac{3 R T}{M}}=\sqrt{\frac{3 k_{B} T}{\mu}}$ square both sides $\quad \frac{3 R T}{M}=\frac{3 k_{B} T}{\mu}$
Get rid of the temperature and the $3: \frac{R}{M}=\frac{k_{B}}{\mu} \quad$ Solve for $k_{B}$
M is the molecular mass which is: $\quad M=\mu N_{A}$
(the mass of a single molecule multiplied by Avogadro's number)
$k_{B}=\frac{\mu R}{M}=\frac{\nless R}{\nless N_{A}}$ and we get $\quad k_{B}=\frac{R}{N_{A}}$
So Boltzmann's constant is simply the ideal gas constant divided by Avogadro's number. Unfortunately, this equation will not be provided to you on the test. You'll just have to know it or be able to develop it.
(4) Explain qualitatively how the model explains the pressure of a gas in terms of collisions with the container walls, and explain how the model predicts that, for fixed volume, pressure must be proportional to temperature.

This is pie. The teacher has explained it all in the AP handouts.
b. You should know how to apply the ideal gas law; and thermodynamics principles so you can:
(1) Relate the pressure and volume of a gas during an isothermal expansion or compression.

This is pretty simple. This is simply the old Boyle's law from chemistry, you know, $P_{1} V_{1}=P_{2} V_{2}$ except you aren't given that equation. You have to derive it from the ideal gas law $P V=n R T$. If the system change is isothermal, then the nRT bit is a constant. This means that PV equals a constant, therefore, no matter what happens to the pressure or the volume, PV is still the same value. This means that $P_{1} V_{1}=P_{2} V_{2}$.

It's really pretty simple. Using the equation is super easy - more pie.
(2) Relate the pressure and temperature of a gas during constant-volume heating or cooling, or the volume and temperature during constant- pressure heating or cooling.

This is another application of the ideal gas law.
If the volume is constant and the pressure and temperature change, use the ideal gas law:

$$
P V=n R T \quad \frac{P}{T}=\frac{n R}{V}
$$

This means that $\frac{P}{T}$ is equal to a constant and $\frac{n R}{V}$ is a constant, so $\frac{P_{1}}{T_{1}}=\frac{P_{2}}{T_{2}}$.
Similarly, if the pressure stays constant, one can develop this equation just like we did before.

$$
\frac{V_{1}}{T_{1}}=\frac{V_{2}}{T_{2}}
$$

(3) Calculate the work performed on or by a gas during an expansion or compression at constant pressure.

The work done by an expansion or compression is given by the equation:

$$
W=-p \Delta V
$$

Luckily this equation is one of the given ones. (Whew.)
(4) Understand the process of adiabatic expansion or compression of a gas.

In an adiabatic process no heat enters or leaves the system. It is the teacher's belief that he did a darn good job of explaining the thing in the handout. Please consult it. The main idea is that $\Delta U=W$.
(5) Identify or sketch on a PV diagram the curves that represent each of the above processes.

This is fairly simple stuff. Consult the handout on how to do this.
2. Laws of Thermodynamics
a. You should know how to apply the first law of thermodynamics so you can:
(1) Relate the heat absorbed by a gas, the work performed by the gas, and the internal energy change of the gas for any of the processes above.

The heat absorbed by a gas can be determined by using the following equation: $Q=m c \Delta T$. The work performed by a gas is given by the $W=-p \Delta V$ equation, or else you can find it as the area under a $P$ vs. V curve. The internal energy change of the gas is given by the first law of thermodynamics; $\Delta U=Q+W$.
(2) Relate the work performed by a gas in a cyclic process to the area enclosed by a curve on a PV diagram.

This is a favorite of the test writers. The basic idea is that the area under the curve represents the work for a single step in the cycle. The net work is the area enclosed by the entire curve. You got the chance to do several of these types of problems in the homework. This is an application of the first law and the idea that the work done is equal to $P \Delta V$ and/or the area under the curve of a $P$ vs. V graph.
b. You should understand the second law of thermodynamics, the concept of entropy, and heat engines and the Carnot cycle so you can:
(1) Determine whether entropy will increase, decrease, or remain the same during a particular situation.

The second law says that it is impossible to build a heat engine that can produce work equivalent to the input heat. It also says that heat will always flow from a hot system to a cold system and never the other way 'round. It tells us that some of the heat put into a system must be returned back to the environment as heat - this implies that the efficiency of a system can never be 100\%; it must always be less.

Entropy is a formalized measure of disorder. As disorder increases, entropy increases. The second law says that entropy tends to increase in all natural processes.
(2) Compute the maximum possible efficiency of a heat engine operating between two given temperatures.

To find the maximum possible efficiency we use the ideal efficiency equation. Remember to convert the temperatures to Kelvins.

$$
e_{c}=\frac{T_{H}-T_{C}}{T_{H}}
$$

(3) Compute the actual efficiency of a heat engine.

To find this, use the general efficiency equation for a heat engine.

$$
e=\left|\frac{W}{Q_{H}}\right|
$$

(4) Relate the heats exchanged at each thermal reservoir in a Carnot cycle to the temperatures of the reservoirs.

Well, what the heck does this mean? Let's try this. The greater the difference in temperature between the two reservoirs, the greater will be the amount of heat that is exchanged. So if more heat is exchanged, then the engine will be more efficient. The greater the temperature of the high temperature reservoir, the greater will be the amount of heat that is transferred. Similarly, the lower the low temperature heat reservoir, the greater will be the amount of heat exchanged.

Once again there's a lot of stuff here.
There are all sorts of thermo questions that can be asked. A common thing on the test is to include a thermo question with some other topic. You might use electricity to heat up a resistor that is in water and the test will ask you for the temperature change of the water. Also possible is a mechanical energy to thermal energy type thing.

## AP Question Time: From 1999:

- A cylinder contains 2.0 moles of an ideal monatomic gas that is initially at state $\boldsymbol{A}$ with a volume of 1.0 x $10^{-2} \mathrm{~m}^{3}$ and a pressure of $4.0 \times 10^{5} \mathrm{~Pa}$. The gas is brought isobarically to state $\boldsymbol{B}$, where the volume is 2.0 x $10^{-2} \mathrm{~m}^{3}$. The gas is then brought at constant volume to state $\boldsymbol{C}$, where its temperature is the same as at state $\boldsymbol{A}$. The gas is then brought isothermally back to state $\boldsymbol{A}$.
a. Determine the pressure of the gas at state $\boldsymbol{C}$.


## We need an equation that relates $P$ and V. We start with the good old ideal gas law:

$P V=n R T$ here the $n R T$ part is a constant since the temperature is the same as when it started. So for changing conditions we get:

$$
\begin{aligned}
& P_{1} V_{1}=n R T=P_{2} V_{2} \quad \text { thus } \quad P_{1} V_{1}=P_{2} V_{2} \\
& \qquad P_{2}=\frac{P_{1} V_{1}}{V_{2}}=\frac{\left(4.0 \times 10^{5} \mathrm{~Pa}\right)\left(1.0 \times 10^{-2} \mathrm{~m}^{3}\right)}{\left(2.0 \times 10^{-2} \mathrm{rx}^{3}\right)}=2.0 \times 10^{5} \mathrm{~Pa}
\end{aligned}
$$

b. On the axes below, state $\boldsymbol{B}$ is represented by the point $\boldsymbol{B}$. Sketch a graph of the complete cycle. Label points $\boldsymbol{A}$ and $\boldsymbol{C}$ to represent states $\boldsymbol{A}$ and $\boldsymbol{C}$, respectively.

c. State whether the net work done by the gas during the complete cycle is positive, negative, or zero. Justify your answer.

Positive. The work done in the $A B$ process is the area under the $A B$ curve and is positive since volume is increasing. The work done in the BC process is zero - no change in volume takes place. The work done in the CA process is the area under the curve, and it is negative since volume is decreasing. Add them together and the net work must be positive, since the area under AB is greater than the area under CA.
d. State whether this device is a refrigerator of a heat engine. Justify you answer.

It is a heat Engine. The work done is positive. Heat is absorbed at a higher temperature and exhausted at a lower temperature.

From 2001:


Note: Figures not drawn to scale.

- A cylinder is fitted with a freely moveable piston of area $1.20 \times 10^{-2} \mathrm{~m}^{2}$ and negligible mass. The cylinder below the piston is filled with a gas. At state 1 , the gas has volume $1.50 \times 10^{-3} \mathrm{~m}^{3}$, pressure, $1.02 \times 10^{5} \mathrm{~Pa}$, and the cylinder is in contact with a water bath at a temperature of $0^{\circ} \mathrm{C}$. The gas is then taken through the following four-step process.
- A 2.50 kg metal block is placed on top of the piston, compressing the gas to state 2 , with the gas still at $0^{\circ} \mathrm{C}$.
- The cylinder is then brought in contact with a boiling water bath, raising the gas temperature to $100^{\circ} \mathrm{C}$ at state 3.
- The metal block is removed and the gas expands to state 4 still at $100^{\circ} \mathrm{C}$.
- Finally, the cylinder is again placed in contact with the water bath at $0^{\circ} \mathrm{C}$, returning the system to state 1 .
(a) Determine the pressure of the gas in state 2 .

The pressure at state 2 must be the initial pressure of the gas plus the pressure exerted by the weight of the metal block.
$\Delta P=\frac{F}{A}+P_{i}=\frac{m g}{A}+P_{i}=\frac{2.5 \mathrm{~kg}\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)}{1.20 \times 10^{-2} \mathrm{~m}^{2}}+1.02 \times 10^{5} \mathrm{~Pa}=20.4 \times 10^{2} \mathrm{~Pa}+1.02 \times 10^{5} \mathrm{~Pa}$
$0.0204 \times 10^{5} \mathrm{~Pa}+1.02 \times 10^{5} \mathrm{~Pa}=1.04 \times 10^{5} \mathrm{~Pa}$
(b) Determine the volume of the gas in state 2 .

We know the pressure and temperature of the gas at both states plus we know the initial volume, so we can develop the combined gas law.
$P V=n R T \quad \frac{P V}{T}=n R \quad$ Since the quantity of $\frac{P V}{T}$ equals a constant, we can say: $\frac{p_{1} V_{1}}{T_{1}}=\frac{p_{2} V_{2}}{T_{2}} \quad$ Therefore: $\frac{p_{1} V_{1}}{T_{1}}=\frac{p_{2} V_{2}}{T_{2}} \quad V_{2}=\frac{\left(1.02 \times 10^{5} \mathrm{Ra}\right)\left(1.50 \times 10^{-3} \mathrm{~m}^{3}\right)}{\left(1.04 \times 10^{5} \mathrm{Ra}\right)}=1.47 \times 10^{-3} \mathrm{~m}^{3}$
(c) Indicate below whether the process from state 2 to state 3 is isothermal, isobaric, or adiabatic.
$\qquad$ Isothermal
 Isobaric $\qquad$ Adiabatic

Explain your reasoning.
The process is not isothermal because the temperature is going up. It is not adiabatic because the system is absorbing heat from the boiling water bath. Therefore, it must be isobaric. (constant pressure)
(d) Is the process from state 4 to state 1 isobaric?
 Yes $\qquad$ No

Explain your reasoning.
This is the reverse process of state $2-3$, which was isobaric, so this one ought to be the same
(e) Determine the volume of the gas in state 4 .

Using the assumption that the process is isobaric, the pressure is $1.02 \times 10^{5} \mathrm{~Pa}$, the same as in state 1 . So:

$$
\frac{p_{1} V_{1}}{T_{1}}=\frac{p_{2} V_{2}}{T_{2}} \quad V_{2}=\frac{p_{1} V_{1} T_{2}}{p_{2} T_{1}} \quad V_{2}=\frac{\left(1.50 \times 10^{-3} \mathrm{~m}^{3}\right) 373 \mathrm{~K}}{273 \mathrm{~K}}=2.05 \times 10^{-3} \mathrm{~m}^{3}
$$

From 1995:

- One mole of an ideal monatomic gas is taken through the cycle $\boldsymbol{a b c a}$ shown on the diagram. State $\boldsymbol{a}$ has volume $\boldsymbol{V}_{\boldsymbol{a}}=17 \times 10^{-3}$ cubic meter and pressure $\boldsymbol{P}_{\boldsymbol{a}}=1.2 \times 10^{5}$ pascals, and state $\boldsymbol{c}$ has volume $\boldsymbol{V}_{\boldsymbol{c}}=51 \times 10^{-3}$ cubic meter. Process calies along the 250 K isotherm. The molar heat capacities for the gas are $\boldsymbol{C}_{\boldsymbol{p}}=20.8$ $\mathrm{J} / \mathrm{mol} \mathrm{K}$, and $\boldsymbol{C}_{\boldsymbol{v}}=12.5 \mathrm{~J} / \mathrm{mol} \mathrm{K}$.

Determine each of the following.
a. The temperature $\boldsymbol{T}_{\boldsymbol{b}}$ of state $\boldsymbol{b}$.

$$
p V=n R T \quad T=\frac{p V}{n R}=\frac{\left(1.2 \times 10^{5} \frac{\mathrm{~K}}{\mathrm{~m}^{2}}\right)\left(51 \times 10^{-3} \mathrm{~m}^{3}\right)}{(1 \mathrm{mql})\left(8.31 \frac{\mathrm{~N} \cdot \mathrm{M}}{\mathrm{mql} \cdot \mathrm{~K}}\right)}=736 \mathrm{~K}
$$

b. The heat $\boldsymbol{Q}_{\boldsymbol{a} \boldsymbol{b}}$ added to the gas during process $\boldsymbol{a} \boldsymbol{b}$.
$Q_{a b}=n c \Delta T=(1 \quad \mathrm{moX})\left(20.8 \frac{\mathrm{~J}}{\mathrm{moX} \cdot \mathrm{K}}\right)(736 \mathrm{~K}-250 \mathrm{~K})=10100 \mathrm{~J}$
c. The change in internal energy $\boldsymbol{U}_{\boldsymbol{b}}-\boldsymbol{U}_{\boldsymbol{a}}$.
$\Delta U=Q-W \quad W=p \Delta V \quad \Delta U=Q-p \Delta V$
$\Delta U=10100 \mathrm{~J}-\left(1.2 \times 10^{5} \mathrm{~Pa}\right)\left(51 \times 10^{-3} \mathrm{~m}^{3}-17 \times 10^{-3} \mathrm{~m}^{3}\right)=6020 \mathrm{~J}$
d. The work $\boldsymbol{W}_{\boldsymbol{b} \boldsymbol{c}}$ done by the gas on its surroundings during process $\boldsymbol{b c}$.

## Work is a force acting through a distance -- distance is zero here.

$$
W=p \Delta V \quad \text { no volume change } W=0
$$

The net heat added to the gas for the entire cycle is 1,800 joules. Determine each of the following.
e. The net work done by the gas on its surroundings for the entire cycle.
$\Delta U=Q-W \quad \Delta U=0 \quad$ Starts at $a$ and ends at a

$$
W_{\text {net }}=Q_{\text {net }} \quad W=1800 \mathrm{~J}
$$

e. The efficiency of a Carnot engine that operates between the maximum and minimum temperatures in this cycle.

$$
e_{c}=\frac{T_{H}-T_{C}}{T_{H}}=\frac{736 \mathrm{~K}-250 \mathrm{~K}}{736 \mathrm{~K}}=0.66 \text { or } 66 \%
$$

From 1996:

- The inside of the cylindrical can shown below has cross-sectional area $0.005 \mathrm{~m}^{2}$ and length 0.15 m . The can is filled with an ideal gas and covered with a loose cap. The gas is heated to 363 K and some is allowed to escape from the can so that the remaining gas reaches atmospheric pressure $\left(1.0 \times 10^{5} \mathrm{~Pa}\right)$. The cap is now tightened, and the gas is cooled to 298 K .

a. What is the pressure of the cooled gas?

Okay, let's look at the ideal gas law first. We can figure out how much gas we have just before we begin to cool it off.

$$
p V=n R T \quad n=\frac{P V}{R T}
$$

The number of moles doesn't change and the volume doesn't change when it is cooled. So we can set the initial and final conditions equal to each other.
$n=\frac{p_{1} \bigvee \mathbb{K}}{R T_{1}}=\frac{p_{2} \bigvee \mathbb{K}}{R T_{2}} \quad \frac{p_{1}}{T_{1}}=\frac{p_{2}}{T_{2}} \quad p_{2}=\frac{p_{1} T_{2}}{T_{1}}$
$P_{2}=\frac{P_{1} T_{2}}{T_{1}}=\frac{1.0 \times 10^{5} \mathrm{~Pa}(298 \mathrm{~K})}{363 \mathrm{~K}}=0.82 \times 10^{5} \mathrm{~Pa}=8.21 \times 10^{4} \mathrm{~Pa}$
b. Determine the upward force exerted on the cap by the cooled gas inside the can.
$p=\frac{F}{A} \quad F=p A=\left(8.2 \times 10^{4} \frac{N}{x^{2}}\right)\left(0.005 x^{2}\right)=410 \mathrm{~N}$
If the cap develops a leak, how many moles of air would enter the can as it reaches a final equilibrium at 298 K and atmospheric pressure? (Assume that air is an ideal gas.)

This is a bit of work. Nothing we can't do, however.
$p V=n R T \quad n=\frac{p V}{R T}$
The inelegant way to do this is to find the number of moles when the lid was capped, then find the number of moles after it was opened. The difference is the number of extra moles of air that entered the can.
$n_{1}=\frac{p V}{R T}=\frac{1.0 \times 10^{5} \frac{K}{\mathrm{~K}^{2}}\left(0.005 \mathrm{~K}^{2}\right)(0.15 \mathrm{Kq})}{8.31 \frac{\mathrm{~K} \cdot \mathrm{~K}}{\mathrm{~mol} \cdot \mathrm{~K}}(363 \mathrm{~K})} \quad n_{1}=0.000000249 \times 10^{5} \mathrm{~mol}=2.49 \times 10^{-2} \mathrm{~mol}$
$n_{2}=\frac{p V}{R T}=\frac{1.0 \times 10^{5} \frac{\mathrm{~K}}{\mathrm{~m}^{2}}\left(5.0 \times 10^{-3} \mathrm{~m}^{2}\right)\left(1.5 \times 10^{-1} \text { 双 }\right)}{8.31 \frac{\mathrm{~K} \cdot \mathrm{~K}}{\mathrm{~mol} \cdot \mathrm{~K}}\left(2.98 \times 10^{2} \mathrm{~K}\right)}$
$n_{2}=0.3029 \times 10^{-1} \mathrm{~mol}=3.03 \times 10^{-2} \mathrm{~mol}$
$\Delta n=n_{2}-n_{1}=\left(3.03 \times 10^{-2} \mathrm{~mol}\right)-\left(2.49 \times 10^{-2} \mathrm{~mol}\right)=0.54 \times 10^{-2} \mathrm{~mol}$

- Students are designing an experiment to demonstrate the conversion of mechanical energy into thermal energy. They have designed the apparatus shown in the figures below. Small lead beads of total mass $\boldsymbol{M}$ and specific heat $\boldsymbol{c}$ fill the lower hollow sphere. The valves between the spheres and the hollow tube can be opened or closed to control the flow of the lead beads. Initially both valves are open.

a. The lower valve is closed and a student turns the apparatus $180^{\circ}$ about a horizontal axis, so that the filled sphere is now on top. This elevates the center of mass of the lead beads by a vertical distance $\boldsymbol{h}$. What minimum amount of work must the student do to accomplish this?


## Work is a change in energy. Gravitational PE is changing.

$$
W=m g h \quad W=M g h
$$

b. The valve is now opened and the lead beads tumble down the hollow tube into the other hollow sphere. If all of the gravitational potential energy is converted into thermal energy in the lead beads, what is the temperature increase of the lead?

$$
Q=U_{g} \quad m c \Delta T=m g h
$$

$$
\Delta T=\frac{g h}{c}
$$

c. The values of $\boldsymbol{M}, \boldsymbol{h}$, and $\boldsymbol{c}$ for the student's apparatus are $\boldsymbol{M}=3.0 \mathrm{~kg}, \boldsymbol{h}=2.00 \mathrm{~m}$ and $\boldsymbol{c}=128 \mathrm{~J} /(\mathrm{kg} \cdot \mathrm{K})$. The students measure the initial temperature of the lead beads and then conduct 100 repetitions of the "elevate and drain" process. Again, assume that all of the gravitational potential energy is converted into thermal energy in the lead beads. Calculate the theoretical cumulative temperature increase after the 100 repetitions.

$$
\Delta T=\frac{\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(2.00 \mathrm{~m})}{\left(128 \frac{\mathrm{~kg} \cdot \mathrm{~m} \cdot}{\mathrm{~s}^{2} \cdot \mathrm{~kg} \cdot \mathrm{~K}}\right)}=0.153 \mathrm{~K}
$$

$$
(0.153 \mathrm{~K})(100 \text { rep })=15.3 \mathrm{~K}
$$

d. Suppose that the experiment were conducted using smaller reservoirs, so that $\boldsymbol{M}$ was one-tenth as large (but $\boldsymbol{h}$ was unchanged). Would your answers to parts (b) and (c) be changed? If so, in what way, and why? If not, why not?

## No, mass has no effect - it cancels out.

e. When the experiment is actually done, the temperature increase is less than calculated in part (c). Identify a physical effect that might account for this discrepancy and explain why it lowers the temperature.

Some of the energy is converted to other things besides heat. Air resistance, sound, etc. Slows the beads reducing their acceleration causing the resulting temperature to be low.

From 1997:

- Three identical resistors, each of resistance $30 \Omega$ are connected in a circuit to heat water in a glass beaker. $24 V$ battery with negligible internal resistance provides the power.
a. The three resistors may be connected in series or in parallel.
i. If they are connected in series, what power is developed in the circuit?

$$
\begin{aligned}
& R_{s}=\sum R_{i}=3(30 \Omega)=90 \Omega \\
& I=\frac{V}{R} P=I V \quad P=\frac{V^{2}}{R}=\frac{(24 V)^{2}}{90 \Omega}=6.4 \mathrm{~W}
\end{aligned}
$$

ii. If they are connected in parallel, what power is developed in the circuit?

$$
\begin{aligned}
& \frac{1}{R_{p}}=\frac{1}{\sum R_{p}}=\frac{1}{30 \Omega}+\frac{1}{30 \Omega}+\frac{1}{30 \Omega} \quad R_{p}=10 \Omega \\
& P=\frac{V^{2}}{R}=\frac{(24 \mathrm{~V})^{2}}{10 \Omega}=57.6 \mathrm{~W}
\end{aligned}
$$

b. Using the battery and one or more of the resistors, design a circuit that will heat the water at the fastest rate when the resistor(s) are placed in the water. Include an ammeter to measure the current in the circuit and a voltmeter to measure the total potential difference of the circuit. Assume the wires are insulated and have no resistance. Draw a diagram of the circuit in the box below, using the following symbols to represent the components in your diagram.


Draw your diagram in this box only.

c. The resistor(s) in the circuit in part (b) are now immersed in a 0.5 kg sample of water, which is initially at 298 K . The specific heat of water is $4,200 \mathrm{~J} / \mathrm{kg} \bullet \mathrm{K}$. Assume that all of the heat produced is absorbed by the water.
i. Calculate the amount of time it takes for the water to begin to boil.

$$
\begin{aligned}
& Q=m c \Delta T \quad Q=(0.5 \mathrm{~kg})\left(4200 \frac{J}{\mathrm{~kg} \cdot \mathrm{~K}}\right)(373 \mathrm{~K}-298 \mathrm{~K})=157500 \mathrm{~J} \\
& P=\frac{W}{t} \quad t=\frac{W}{P}=157500 \mathrm{X}\left(\frac{1}{57.6 \frac{\mathrm{X}}{\mathrm{~S}}}\right)=2734 \mathrm{~s}
\end{aligned}
$$

ii. Under actual experimental conditions, would the time taken for the water to boil be longer or shorter than the calculated time in part (c, i)? Justify your answer.

## Longer. Some heat is lost to the environment.

d. As the circuit continues to provide energy to the water, vapor is formed at the same temperature as the boiling water. Where has the energy used to boil the water gone?

## A latent heat of evaporation is required to overcome the strength of the hydrogen bond intermolecular forces.

From 1992:

- A portion of an electric circuit connected to a 40 -ohm resistor is embedded in 0.20 kilogram of a solid substance in a calorimeter. The external portion of the circuit is connected to a 60 -volt power supply, as shown.

a. Calculate the current in the resistor.

$$
V=I R \quad I=\frac{V}{R}=\frac{60 \mathrm{~V}}{40 \Omega}=1.5 \mathrm{~A}
$$

b. Calculate the rate at which heat is generated in the resistor.

Heat is energy, and the rate of energy is power

$$
P=I V \quad P=(1.5 \mathrm{~A})(60 \mathrm{~V})=90 \mathrm{~W}
$$

c. Assuming that all of the heat generated by the resistor is absorbed by the solid substance, and that it takes 4 minutes to raise the temperature of the substance from $20^{\circ} \mathrm{C}$ to $80^{\circ} \mathrm{C}$, calculate the specific heat of the substance.
$P=\frac{W}{t} \quad W=P t=\left(90 \frac{J}{\mathrm{~S}}\right) 4 \mathrm{~min}\left(\frac{60 \mathrm{~s}}{1 \mathrm{~min}}\right)=21600 \mathrm{~J}$

## Work is the change in energy

$Q=m c \Delta T \quad c=\frac{Q}{m \Delta T}=\frac{21600 \mathrm{~J}}{(0.2 \mathrm{~kg})\left(80^{\circ} \mathrm{C}-20^{\circ} \mathrm{C}\right)}=\quad 1800 \frac{\mathrm{~J}}{\mathrm{~kg} \cdot{ }^{\circ} \mathrm{C}}$
d. At $80^{\circ} \mathrm{C}$ the substance begins to melt. The heat of fusion of the substance is $1.35 \times 10^{5}$ joules per kilogram. How long after the temperature reaches $80^{\circ} \mathrm{C}$ will it take to melt all of the substance?

$$
\begin{aligned}
& Q=m L_{F}=(0.2 \mathrm{kQ})\left(1.35 \times 10^{5} \frac{\mathrm{~J}}{\mathrm{kQ}}\right)=27000 \mathrm{~J} \quad P=\frac{W}{t} \\
& t=\frac{W}{P}=\frac{27000 \mathrm{X}}{90 \frac{\mathrm{X}}{\mathrm{~s}}}=300 \times\left(\frac{1 \mathrm{~min}}{60 \mathrm{~K}}\right)=55 \mathrm{~min}
\end{aligned}
$$

e. Draw a graph of the heating curve for the substance on the axes below, showing the temperature as a function of time until all of the solid has melted. Be sure to put numbers and units on the time scale.


