How do we go from our old equation to the new equation?

Δ*Eg* = *mg*Δ*h*

The old equation is hidden in the new equation.

|  |  |
| --- | --- |
| *Ep* = −  | *Gm*1*m*2 |
| *r* |

Let me show you.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Δ*Eg* = |   | *Ef* |  −  | *Ei* |
|   |
| Δ*Eg* = |   | *Eg*(*r* + Δ*h*) |  −  | *Eg*(*r*) |
|   |
| Δ*Eg* = |  −  | *Gm*1*m*2 |  +  | *Gm*1*m*2 |
| *r* + Δ*h* | *r* |

Combine terms over a common denominator.

|  |  |
| --- | --- |
| Δ*Eg* =  | *Gm*1*m*2((*r* + Δ*h*) − *r*) |
| *r*(*r* + Δ*h*) |
| Δ*Eg* =  | *Gm*1*m*2Δ*h* |                 |
| *r*(*r* + Δ*h*) |   |

Multiply by "one".

|  |  |  |  |
| --- | --- | --- | --- |
| Δ*Eg* =  | *Gm*1*m*2Δ*h* |   | *r* |
| *r*(*r* + Δ*h*) |   | *r* |

Swap terms in the denominators.

|  |  |  |  |
| --- | --- | --- | --- |
| Δ*Eg* =  | *Gm*1*m*2Δ*h* |   | *r* |
| *r*2 |   | *r* + Δ*h* |

Factor some stuff out of the numerator.

|  |  |  |  |
| --- | --- | --- | --- |
| Δ*Eg* = *m*2*h*  | *Gm*1 |   | *r* |
| *r*2 |   | *r* + Δ*h* |

Do you see it? If *r* is the radius of the earth, *m*1 is the mass of the earth, and *m*2 is the mass of something being lifted, then …

|  |  |
| --- | --- |
| *g* =  | *Gm*1 |
| *r*2 |

is the acceleration due to gravity on the earth's surface. Making this substitution (and dropping the subscript, since we only have one mass left), we get …

|  |  |
| --- | --- |
| Δ*Eg* = *mg*Δ*h*  | *r* |
| *r* + Δ*h* |

The first part of this expression is our old friend, the original equation for gravitational potential energy. The second term is a correction factor. For ordinary heights, this term is essentially one. Let's confirm this using a really high height — the top of the spire on the Burj Khalifa in the United Arab Emirates (818 m).

|  |  |  |  |
| --- | --- | --- | --- |
| *r* |  =  | 6,371,000 m |  = 0.999872 |
| *r* + Δ*h* | 6,371,000 m + 818 m |

The engineers who designed the Burj would have an error in the fourth decimal place of their calculations. This deviation is probably smaller than the uncertainty in the mass of the girders used to construct this building, which is why Δ*Ug* = *mg*Δ*h* is totally acceptable for most down-to-earth applications

Now let's try something astronomical. Can Δ*Ug* = *mg*Δ*h* be used to measure the gravitational potential energy of the moon? The earth–moon distance (384,400,000 m) is measured from the center of the earth, not it's surface. In this case, *r* + Δ*h* will actually be a difference in two numbers.

|  |  |  |  |
| --- | --- | --- | --- |
| *r* |  =  | 6,371,000 m |  =  0.016853 |
| *r* + Δ*h* | 384,400,000 m − 6,371,000 m |

This number is obviously closer to zero than to one, which is why …

|  |  |
| --- | --- |
| *Eg* = −  | *Gm*1*m*2 |
| *r* |

is used for astronomical applications.