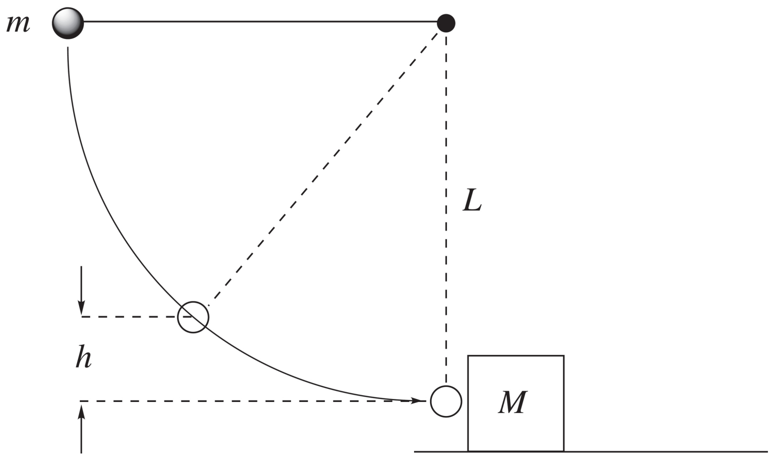
Section II: Free Response

1. A steel ball of mass m is fastened to a light cord of length L and released when the cord is horizontal. At the bottom of its path, the ball strikes a hard plastic block of mass $M = 4m$, initially at rest on a frictionless surface. The collision is elastic.

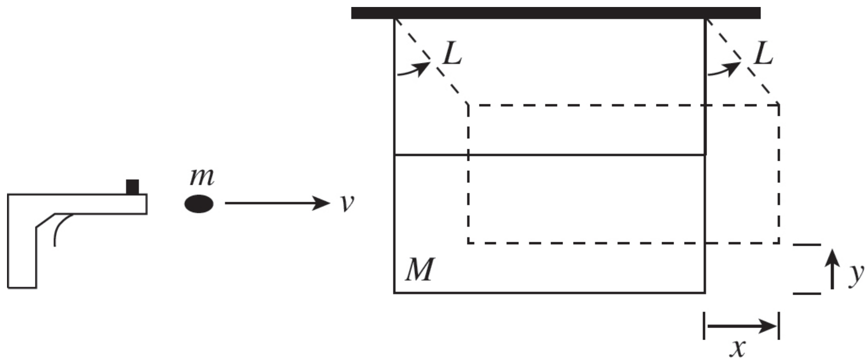


(a) Find the tension in the cord when the ball's height above its lowest position is $\frac{1}{2}L$. Write your answer in terms of m and g.

(b) Find the speed of the block immediately after the collision.

(c) To what height h will the ball rebound after the collision?

2. A *ballistic pendulum* is a device that may be used to measure the muzzle speed of a bullet. It is composed of a wooden block suspended from a horizontal support by cords attached at each end. A bullet is shot into the block, and as a result of the perfectly inelastic impact, the block swings upward. Consider a bullet (mass m) with velocity v as it enters the block (mass M). The length of the cords supporting the block each have length L. The maximum height to which the block swings upward after impact is denoted by y, and the maximum horizontal displacement is denoted by x.



(a) In terms of m, M, g, and y, determine the speed v of the bullet.

(b) What fraction of the bullet's original kinetic energy is lost as a result of the collision? What happens to the lost kinetic energy?

(c) If y is very small (so that y^2 can be neglected), determine the speed of the bullet in terms of m, M, g, x, and L.

(d) Once the block begins to swing, does the momentum of the block remain constant? Why or why not?

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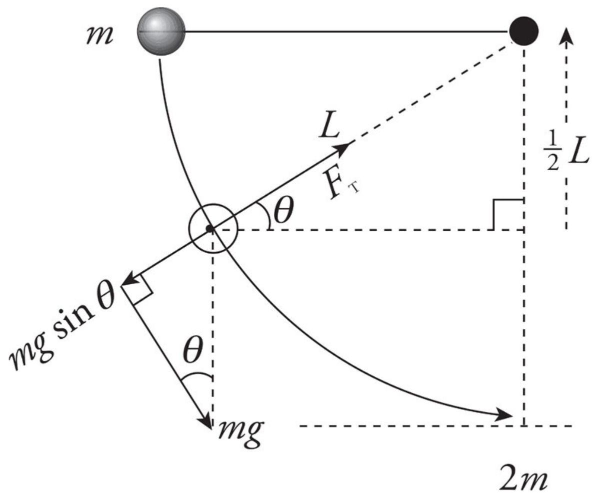
1. (a)

First, draw a free-body diagram:



The net force toward the center of the steel ball's circular path provides the centripetal force. From the geometry of the diagram, you get

$$F_T - mg\sin\theta = \frac{mv^2}{L} \qquad (*)$$

In order to determine the value of mv^2, use Conservation of Mechanical Energy:

$$K_i + U_i = K_f + U_f$$
$$0 + mgL = \frac{1}{2}mv^2 + mg(\frac{1}{2}L)$$
$$\frac{1}{2}mgL = \frac{1}{2}mv^2$$
$$mgL = mv^2$$

Substituting this result into Equation (*), you get

$$F_T - mg\sin\theta = \frac{mgL}{L}$$
$$F_T = mg(1+\sin\theta)$$

Now, from the free-body diagram, $\sin\theta = \frac{1}{2}L/L = \frac{1}{2}$, so

$$F_T = mg(1+\frac{1}{2}) = \frac{3}{2}mg$$

(b)

Apply Conservation of Energy to find the speed of the ball just before impact:

$$K_i + U_i = K_f + U_f$$
$$0 + mgL = \frac{1}{2}mv^2 + 0$$
$$v = \sqrt{2gL}$$

Now use Conservation of Linear Momentum and conservation of kinetic energy for the elastic collision to derive the expressions for the speeds of the ball, v_1, and the block, v_2, immediately after the collision. Applying Conservation of Linear Momentum yields

$$m\sqrt{2gL} = mv_1 + 4mv_2$$

$$\sqrt{2gL} = v_1 + 4v_2$$

$$v_1 = \sqrt{2gL} - 4v_2$$

Applying the conservation of kinetic energy for the elastic collision yields

$$\frac{1}{2}m\left(\sqrt{2gL}\right)^2 = \frac{1}{2}\,mv_1^2 + \frac{1}{2}\,(4m)v_2^2$$

$$2gL = v_1^2 + (4m)v_2^2$$

Plugging in the expression for v_1 from the Conservation of Linear Momentum yields

$$2gL = \left(\sqrt{2gL} - 4v_2\right)^2 + 4v_2^2$$

$$2gL = 2gL - 8\sqrt{2gL}v_2 + 16v_2^2 + 4v_2^2$$

$$0 = 20v_2^2 - 8\sqrt{2gL}v_2$$

$$0 = 2v_2(10v_2 - 4\sqrt{2gL})$$

$$0 = 10v_2 - 4\sqrt{2gL}$$

$$10v_2 = 4\sqrt{2gL}$$

$$v_2 = \frac{2\sqrt{2}}{5}\sqrt{gL}$$

(c)

Using the velocity of the block immediately after the collision, v_2, solve for the velocity of the ball immediately after the collision:

$$v_1 = \sqrt{2gL} - 4v_2$$

$$v_1 = \sqrt{2gL} - 4\left(\frac{2\sqrt{2}}{5}\sqrt{gL}\right)$$

$$v_1 = \sqrt{2gL} - \frac{4 \cdot 2}{5}\sqrt{2gL}$$

Factor out $\sqrt{2gL}$ to get

$$v_1 = \sqrt{2gL}\left(1 - \frac{8}{5}\right)$$

$$v_1 = -\frac{3}{5}\sqrt{2gL}$$

Now, apply Conservation of Mechanical Energy to find

$$K_i + U_i = K_f + U_f$$
$$\frac{1}{2}mv_1'^2 + 0 = 0 + mgh$$
$$h = \frac{v_1'^2}{2g} = \frac{\left(\frac{3}{5}\sqrt{2gL}\right)^2}{2g} = \frac{9}{25}L$$

2. (a)

By Conservation of Linear Momentum, $mv = (m + M)v'$, so $v' = \frac{mv}{m+M}$

Now, by Conservation of Mechanical Energy,

$$K_i + U_i = K_f + U_f$$
$$\frac{1}{2}(m+M)v'^2 + 0 = 0 + (m+M)gy$$
$$\frac{1}{2}v'^2 = gy$$
$$\frac{1}{2}\left(\frac{mv}{m+M}\right)^2 = gy$$
$$v = \frac{m+M}{m}\sqrt{2gy}$$

(b)

Use the result derived in part (a) to compute the kinetic energy of the block and bullet immediately after the collision:

$$K' = \frac{1}{2}(m+M)v'^2 = \frac{1}{2}(m+M)\left(\frac{mv}{m+M}\right)^2 = \frac{1}{2}\frac{m^2v^2}{m+M}$$

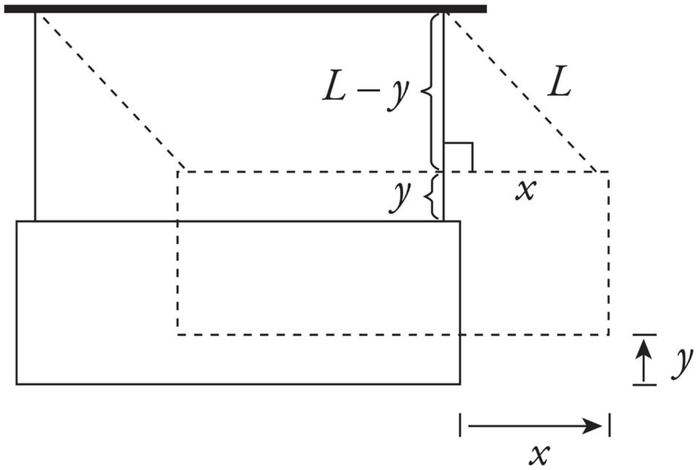
Since $K = \frac{1}{2}mv^2$, the difference is

$$\begin{aligned}\Delta K = K' - K &= \frac{1}{2}\frac{m^2v^2}{m+M} - \frac{1}{2}mv^2 \\ &= \frac{1}{2}mv^2\left(\frac{m}{m+M} - 1\right) \\ &= K\left(\frac{-M}{m+M}\right)\end{aligned}$$

Therefore, the fraction of the bullet's original kinetic energy that was lost is $M/(m + M)$. This energy is manifested as heat (the bullet and block are warmer after the collision than before), and some was used to break the intermolecular bonds within the wooden block to allow the bullet to penetrate.

(c)

From the geometry of the diagram,



the Pythagorean Theorem implies that $(L - y)^2 + x^2 = L^2$. Therefore,

$$L^2 - 2Ly + y^2 + x^2 = L^2 \quad \Rightarrow \quad y = \frac{x^2}{2L}$$

(where you have used the fact that y^2 is small enough to be neglected). Substituting this into the result of part (a), derive the following equation for the speed of the bullet in terms of x and L instead of y:

$$v = \frac{m+M}{m}\sqrt{2gy} = \frac{m+M}{m}\sqrt{2g\frac{x^2}{2L}} = \frac{m+M}{m}x\sqrt{\frac{g}{L}}$$

(d)

No; momentum is conserved only when the net external force on the system is zero (or at least negligible). In this case, the block and bullet feel a net nonzero force that causes it to slow down as it swings upward. Since its speed is decreasing as it swings upward, its linear momentum cannot remain constant.