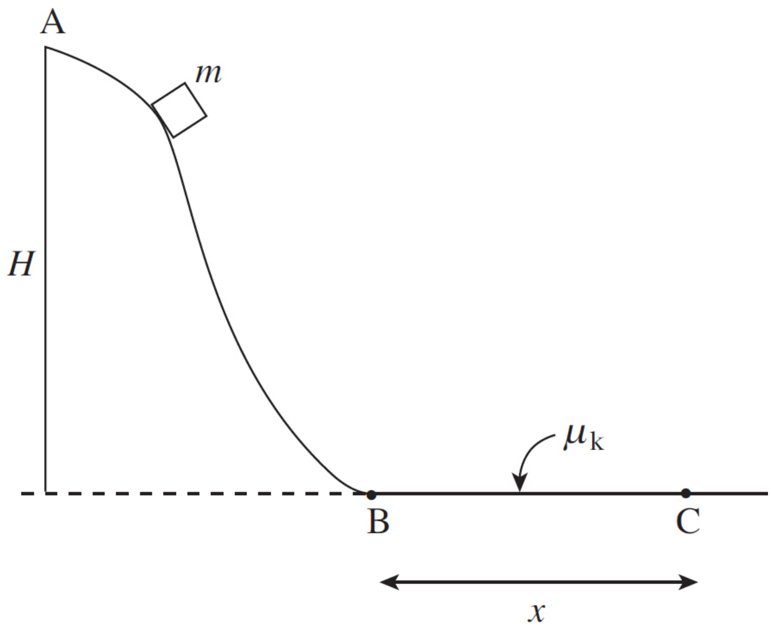
Section II: Free Response

1. A box of mass m is released from rest at Point A, the top of a long, frictionless slide. Point A is at height H above the level of Points B and C. Although the slide is frictionless, the horizontal surface from Point B to C is not. The coefficient of kinetic friction between the box and this surface is μ_k, and the horizontal distance between Point B and C is x.



(a) Find the speed of the box when its height above Point B is $\frac{1}{2}H$.

(b) Find the speed of the box when it reaches Point B.

(c) Determine the value of μ_k so that the box comes to rest at Point C.

(d) Now assume that Points B and C were not on the same horizontal level. In particular, assume that the surface from B to C had a uniform upward slope so that Point C were still at a horizontal distance of x from B but now at a vertical height of y above B. Answer the question posed in part (c).

(e) If the slide were not frictionless, determine the work done by friction as the box moved from Point A to Point B if the speed of the box as it reached Point B were half the speed calculated in part (b).

2. A student uses a digital camera and computer to collect the following data about a ball as it slides down a curved frictionless track. The initial release point is 1.5 meters above the ground and the ball is released from rest. He prints up the following data and then tries to analyze it.

time (s)	velocity (m/s)
0.00	0.00
0.05	1.41
0.10	2.45
0.15	3.74
0.20	3.74

0.25	3.46
0.30	3.16
0.35	2.83
0.40	3.46
0.45	4.24
0.50	4.47

(a) Based on the data, what are the corresponding heights for each data point?

(b) What time segment experiences the greatest acceleration and what is the value of this acceleration?

(c) How would the values change if the ball were replaced by an identical ball with double the mass?

3. A car with a mass of 800 kg is traveling with an initial speed of 10 m/s. When the brakes of the car are applied, the car starts to skid, and it experiences a frictional force with $\mu_k = 0.2$.

(a) What is the skidding distance of the car?

(b) How would the skidding distance change if the initial speed of the car were doubled?

(c) How would the skidding distance change if the initial mass of the car were doubled?

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1. (a)

Applying Conservation of Energy,

$$K_A + U_A = K_{\text{at } H/2} + U_{\text{at } H/2}$$
$$0 + mgH = \frac{1}{2}mv^2 + mg(\frac{1}{2}H)$$
$$\frac{1}{2}mgH = \frac{1}{2}mv^2$$
$$v = \sqrt{gH}$$

(b)

Applying Conservation of Energy again,

$$K_A + U_A = K_B + U_B$$
$$0 + mgH = \frac{1}{2}mv_B^2 + 0$$
$$v_B = \sqrt{2gH}$$

(c)

By the Work–Energy Theorem, you want the work done by friction to be equal (but opposite) to the kinetic energy of the box at Point B:

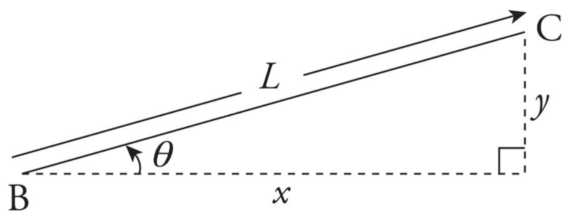
$$W = \Delta K = \frac{1}{2}m(v_C^2 - v_B^2) = -\frac{1}{2}mv_B^2 = -\frac{1}{2}m(\sqrt{2gH})^2 = -mgH$$

Therefore,

$W = -mgH \quad \Rightarrow \quad -F_f x = -mgH \quad \Rightarrow \quad -\mu_k mgx = -mgH \quad \Rightarrow \quad \mu_k = H/x$

(d)

Apply Conservation of Energy (including the negative work done by friction as the box slides up the ramp from B to C):



$$K_B + U_B + W_f = K_C + U_C$$
$$\frac{1}{2}m(\sqrt{2gH})^2 + 0 - F_f L = 0 + mgy$$
$$mgH + 0 - F_f L = 0 + mgy$$
$$mg(H - y) - (\mu_k mg\cos\theta)(L) = 0$$
$$\mu_k = \frac{H - y}{L\cos\theta} = \frac{H - y}{x}$$

(e)

The result of part (b) reads $v_B = \sqrt{2gH}$. Therefore, by Conservation of Mechanical Energy (with the work done by the frictional force on the slide included), you get

$$\begin{aligned} K_A + U_A + W_f &= K_B' + U_B \\ 0 + mgH + W_f &= \frac{1}{2}m\left(\frac{1}{2}v_B\right)^2 + 0 \\ mgH + W_f &= \frac{1}{2}m\left(\frac{1}{2}\sqrt{2gH}\right)^2 \\ mgH + W_f &= \frac{1}{4}mgH \\ W_f &= -\frac{3}{4}mgH \end{aligned}$$

2. (a)

Using Conservation of Energy $K_i + U_i = K_f + U_f$ and $v_i = 0$, this becomes $U_i = K_f + U_f$ or $K_f = U_i - U_f$. This is equivalent to $\frac{1}{2}mv^2 = mgh_i - mgh_f$, which simplifies to $\frac{1}{2}v^2 - gh_i = -gh_f$ or $h_f = h_i - \frac{v^2}{2g}$. Now fill in the table.

Time (s)	Velocity (m/s)	Height (m)
0.00	0.00	1.5
0.05	1.41	1.4
0.10	2.45	1.2
0.15	3.74	0.8
0.20	3.74	0.8
0.25	3.46	0.9
0.30	3.16	1.0
0.35	2.83	1.1
0.40	3.46	0.9
0.45	4.24	0.6
0.50	4.47	0.5

(b)

The greatest acceleration would occur where there is the greatest change in velocity. This occurs between 0.00 and 0.05 seconds. The acceleration during that time interval is given by $a = \frac{v_f - v_i}{t_f - t_i} \Rightarrow \frac{1.41 - 0}{0.05 - 0.00}$ or $a = 28\ \text{m/s}^2$.

(c)

Changing the mass does not affect the time spent falling or the velocity of the object. Thus, a change in mass will not affect the results.

3. (a)

Use the Work–Energy Theorem:

$$W_{\text{total}} = \Delta K$$

The force doing work during the motion is provided by the force of friction:

$$W = F_f \cdot d \cdot \cos\theta = \Delta K$$

$$\mu_k F_N \cdot d \cdot \cos\theta = \frac{1}{2}m(v^2 - v_0^2)$$

As the force of friction is antiparallel to the direction of the displacement, $\theta = 180°$.

$$\mu_k F_N \cdot d \cdot \cos\theta = \frac{1}{2}m(v^2 - v_0^2)$$

$$\mu_k mg \cdot d \cdot \cos\theta = \frac{1}{2}m(v^2 - v_0^2)$$

$$\mu_k g \cdot d \cdot \cos\theta = \frac{(v^2 - v_0^2)}{2}$$

$$d = \frac{(v^2 - v_0^2)}{2\mu_k \cdot g \cdot \cos\theta} = \frac{(0 - 10^2)}{2(0.2) \cdot 10 \cdot \cos 180°} = \frac{-100}{-4} = 25 \text{ m}$$

The skidding distance is 25 m.

(b)

The final equation for the skidding distance is

$$d = \frac{(v^2 - v_0^2)}{2\mu_k \cdot g \cdot \cos\theta}$$

Since the final velocity is 0, the stopping distance is proportional to the initial speed. So if the initial speed is doubled, the skidding distance is quadrupled. This can also be determined by plugging in an initial velocity of 20 m/s:

$$d = \frac{(v^2 - v_0^2)}{2\mu_k \cdot g \cdot \cos\theta} = \frac{(0 - 20^2)}{2(0.2) \cdot 10 \cdot \cos 180°} = \frac{-400}{-4} = 100\ m$$

(c)

Look back at the equation for the skidding distance:

$$d = \frac{(v^2 - v_0^2)}{2\mu_k \cdot g \cdot \cos\theta}$$

This equation does not include mass, so mass does not affect the skidding distance. (Note: While doubling the mass doubles the initial kinetic energy of the car, it also doubles the normal force and thus the frictional force acting on the car.)