Work, Energy, and Power

"Energy cannot be created or destroyed: it can only be changed from one form to another."

—Albert Einstein

Kinematics and dynamics are about change. Simple observations of our environment show us that change is occurring all around us. But what is needed to make an object change, and where did that change go to? It wasn't until more than one hundred years after Newton that the idea of energy became incorporated into physics, but today it permeates every branch of the subject.

ENERGY: AN OVERVIEW

So what is **energy?** How do we determine the energy of a system? These are not easy questions. It is difficult to give a precise definition of energy; there are different forms of energy as a result of different kinds of forces. Energy can come as a result of gravitational force, the speed of an object, stored energy in springs, heat loss to nuclear energy. But one truth remains the same for all of them: the Law of Conservation of Energy. Energy cannot just appear out of nowhere, nor can it disappear in a closed system; it must always take on another form. Force is the agent for this change, energy is the measure of that change, and work is the method of transferring energy from one system to another.

WORK

When you lift a book from the floor, you exert a force on it over a distance, and when you push a crate across a floor, you also exert a force on it over a distance. The application of force over a distance and the resulting change in energy of the system that the force acted on, give rise to the concept of **work.**

> **Work Units**
> The unit for work, the newton-meter (N·m), is renamed a joule and abbreviated as J.
>
> Named after English physicist James Prescott Joule, one joule is the work required to produce one watt of power for one second.

Definition. If a constant force **F** acts over a distance **d** and **F** is parallel to **d,** then the work done by **F** is the product of force and distance:

$$W = \mathbf{F}\mathbf{d}$$

Notice that, although work depends on two vectors **(F** and **d),** work itself is *not* a vector. *Work is a scalar quantity.* However, even being a scalar quantity there exists positive, negative, and zero work.

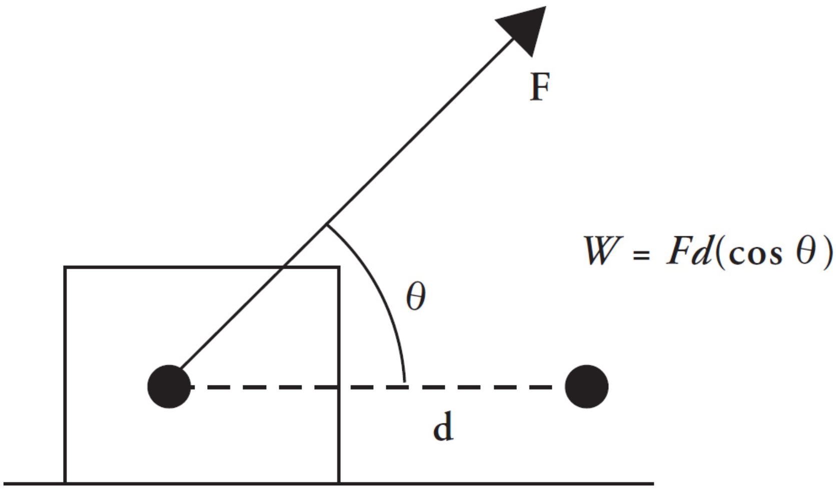
> **Example 1** You slowly lift a book of mass 2 kg at constant velocity a distance of 3 m. How much work did you do on the book?

Solution. In this case, the force you exert must balance the weight of the book (otherwise the velocity of the book wouldn't be constant), so $F = mg$ = (2 kg) (10 m/s^2) = 20 N. Since this force is straight upward and the displacement of the book is also straight upward, **F** and **d** are parallel, so the work done by your lifting force is $W = Fd$ = (20 N)(3 m) = 60 N·m, or 60 J.

> **When the Formula for Work Works**
> $W = Fd \cos\theta$ works only when the forces do not change as the object moves. This also means a constant acceleration is delivered on the mass.

WORK AT AN ANGLE

The previous formula works only when work is done completely parallel to the intended distance of travel. What happens when the force is done at an angle? The formula becomes:



$$W = Fd\,(\cos\theta)$$

Let's compare the work done in a few instances with a force being applied between certain angles:

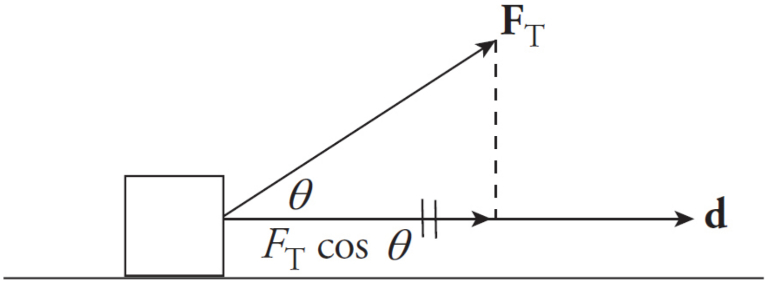
Angle (θ)	$0 \le \theta < 90$	$\theta = 90$	$90 < \theta \le 180$
Cos (θ)	positive	ZERO	negative
Work	positive	ZERO	negative
Speed of Object	increases	constant speed	decreases

Note that when we do positive work, we increase the speed of an object; however, when we do negative work, we slow an object down. This will be important to note when we relate work with kinetic energy and potential energy.

Don't Be Tricked!
A force applied perpendicular to the intended direction of motion always does ZERO work!

Example 2 A 15 kg crate is moved along a horizontal floor by a warehouse worker who's pulling on it with a rope that makes a 30° angle with the horizontal. The tension in the rope is 69 N, and the crate slides a distance of 10 m. How much work is done on the crate by the worker?

Solution. The figure on the next page shows that $\mathbf{F}_T$ and $\mathbf{d}$ are not parallel. It's only the component of the force acting along the direction of motion, $F_T \cos\theta$, that does work.



Therefore,

$$W = (F_\mathrm{T}\cos\theta)d = (69\text{ N}\cdot\cos 30°)(10\text{ m}) = 600\text{ J}$$

Example 3 In the previous example, assume that the coefficient of kinetic friction between the crate and the floor is 0.4.

(a) How much work is done by the normal force?

(b) How much work is done by the friction force?

Solution.

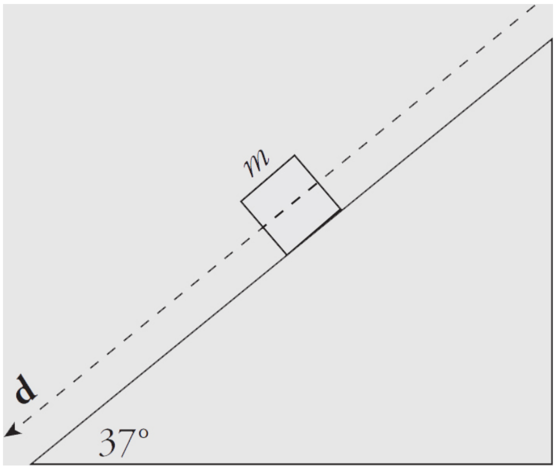
(a) Clearly, the normal force is not parallel to the motion, so we use the general definition of work. Since the angle between $\mathbf{F}_\mathrm{N}$ and **d** is 90° (by definition of *normal*) and cos 90° = 0, the normal force does zero work.

(b) The friction force, $\mathbf{F}_\mathrm{f}$, is also not parallel to the motion; it's *antiparallel*. That is, the angle between $\mathbf{F}_\mathrm{f}$ and **d** is 180°. Since cos 180° = −1, and since the strength of the normal force is $F_\mathrm{n} = F_\mathrm{w} - F_{\mathrm{T,y}} = mg - F_\mathrm{T}\cdot\sin(\theta) = (15\text{ kg})(10\text{ m/s}^2) - (69\text{ N})(1/2) = 115.5\text{ N}$, the work done by the friction force is

$$W = -F_\mathrm{f}d = -\mu_\mathrm{k}F_\mathrm{N}\cdot d = -(0.4)(115.5\text{ N})(10\text{ m}) = -462\text{ J}$$

Antiparallel
When vectors that are parallel but pointing in opposite directions, if these vectors are joined at the tail, they form an angle of 180 degrees.

Example 4 A box slides down an inclined plane (incline angle = 37°). The mass of the block, *m*, is 35 kg, the coefficient of kinetic friction between the box and the ramp, μ_k, is 0.3, and the length of the ramp, *d*, is 8 m.



(a) How much work is done by gravity?

(b) How much work is done by the normal force?

(c) How much work is done by friction?

(d) What is the total work done?

Solution.

(a) Recall that the force that's directly responsible for pulling the box down the plane is the component of the gravitational force that's parallel to the ramp: $F_w \sin\theta = mg \sin\theta$(where θ is the incline angle). This component is parallel to the motion, so the work done by gravity is

$$W_{\text{by gravity}} = (mg \sin\theta)d = (35 \text{ kg})(10 \text{ N/kg})(\sin 37°)(8 \text{ m}) = 1690 \text{ J}$$

Note that the work done by gravity is positive, as we would expect it to be, since gravity is helping the motion. Also, be careful with the angle θ. The general definition of work reads $W = (F \cos\theta)d$, where θ is the angle between **F** and **d.** However, the angle between $\mathbf{F}_w$ and **d** is *not* 37° here, so the work done by gravity is not $(mg \cos 37°)d$. The angle θ used in the calculation above is the incline angle.

(b) Since the normal force is perpendicular to the motion, the work done by this force is zero.

(c) The strength of the normal force is $F_w \cos\theta$ (where θ is the incline angle), so the strength of the friction force is $F_f = \mu_k F_N = \mu_k F_w \cos\theta = \mu_k mg \cos\theta$. Since $\mathbf{F}_f$ is antiparallel to **d,** the cosine of the angle between these vectors (180°) is −1, so the work done by friction is

$$\begin{aligned} W_{\text{by friction}} &= -F_f d = -(\mu_k mg \cos\theta)(d) \\ &= -(0.3)(35 \text{ kg})(10 \text{ N/kg})(\cos 37°)(8 \text{ m}) \\ &= -671 \text{ J} \end{aligned}$$

Note that the work done by friction is negative, as we expect it to be, since friction is opposing the motion.

(d) The total work done is found simply by adding the values of the work done by each of the forces acting on the box:

$$W_{\text{total}} = \Sigma W = W_{\text{by gravity}} + W_{\text{by normal force}} + W_{\text{by friction}} = 1{,}690 + 0 + (-671) = 1{,}019 \text{ J}$$

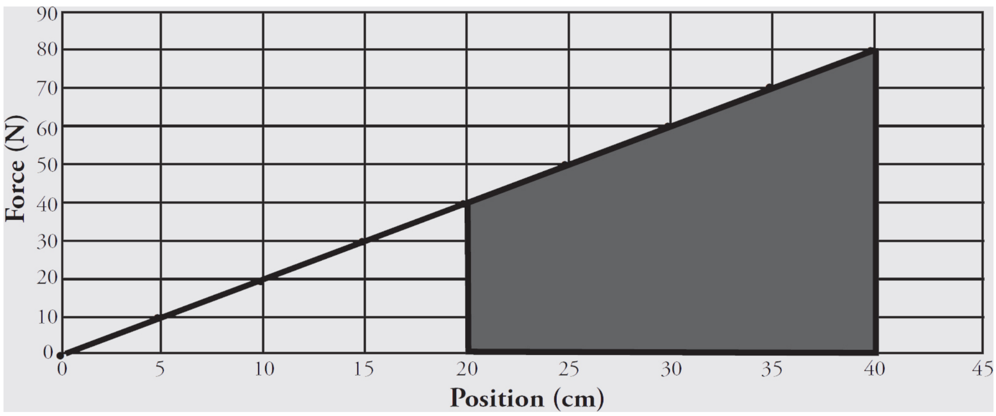
Zero Work
Part (b) of this question is a typical trick question. Just remember, a force applied perpendicular to direction of travel does zero work. Some people think that normal force can never do work, but that's taking things too far. For example, any time you ride up in an elevator, the normal force is doing work.

WORK DONE BY A VARIABLE FORCE

If a force remains constant over the distance through which it acts, then the work done by the force is simply the product of force and distance. However, if the force does not remain constant, then the work done by the force is given by the area under the curve of a force-versus-displacement graph. In physics language, the term "under the curve" really means between the line itself and zero.

Big Idea #3

Example 5 A spring exerts a force as shown on the graph below. How much work is done as the spring stretches from 20 to 40 cm?



Solution. The area under the curve will be equal to the work done. In this case, we have some choices. You may recognize this shape as a trapezoid (it might help to momentarily rotate your head, or this book, 90 degrees) to see this.

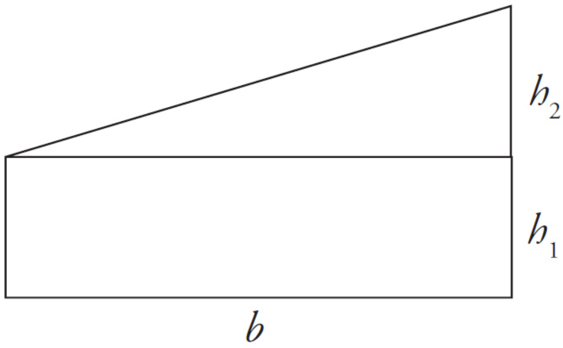
$$A = \frac{1}{2}(b_1 + b_2)h$$

$$A = \frac{1}{2}(40\,\text{N} + 80\,\text{N})(0.20\text{ m})$$

$$= 12\text{ N} \cdot \text{m or } 12\text{ J}$$

Similar to the previous chapter, units for work and energy should be confined to kg, m, and s, which is why we converted here.

An alternative choice is to recognize this shape as a triangle sitting on top of a rectangle. The total area is simply the area of the rectangle plus the area of the triangle.



$$A_{total} = A_{rectangle} + A_{triangle}$$

$$= bh_1 + \frac{1}{2}(bh_2)$$

$$= (0.20 \text{ m})(40\,\text{N} - 0\,\text{N}) + \frac{1}{2}(0.20 \text{ m})(80\,\text{N} - 40\,\text{N})$$

$$= 8 \text{ N} \cdot \text{m} + 4 \text{ N} \cdot \text{m} = 12 \text{ J}$$

KINETIC ENERGY

Consider an object at rest ($v_0 = 0$), and imagine that a steady force is exerted on it, causing it to accelerate. Let's be more specific; let the object's mass be m, and let **F** be the force acting on the object, pushing it in a straight line. The object's acceleration is $a = F/m$, so after the object has traveled a distance Δx under the action of this force, its final speed, v, is given by Big Five #5:

$$v^2 = v_0^2 + 2a(x - x_0) = 2a(x - x_0) = 2\frac{F}{m}(x - x_0) \quad \Rightarrow \quad F(x - x_0) = \frac{1}{2}mv^2$$

But the quantity $F(x - x_0)$ is the work done by the force, so $W = \frac{1}{2}mv^2$. The work done on the object has transferred energy to it, in the amount $\frac{1}{2}mv^2$. The energy an object possesses by virtue of its motion is therefore defined as $\frac{1}{2}mv^2$ and is called **kinetic energy:**

Potential energy comes from things being in positions they don't want to be in. Two electrons don't want to be near each other. A spring doesn't want to be compressed. When a system moves to a more natural configuration (electrons move apart, spring returns to equilibrium, etc.), potential energy is lost.

$$K = \frac{1}{2}mv^2$$

Need for Speed
As we mentioned before, when we do positive work, we increase the speed of an object. As a result, we also increase its kinetic energy. Doing positive work means making a positive change in kinetic energy.

THE WORK–ENERGY THEOREM

In the previous section, we derived kinetic energy from Big Five #5. Let's solve it a different way this time:

$v^2 = v_0^2 + 2ad$ Recall: $a = F/m$

$2(F/m)d = v^2 - v_0^2$ Recall: $W = Fd$

$W = (1/2)mv^2 - (1/2)mv_0^2$

$W = K_{\text{final}} - K_{\text{initial}}$

$$W_{\text{total}} = \Delta K$$

One of the questions posed at the beginning of this chapter was "How does a system gain or lose energy?" The **Work–Energy Theorem** begins to answer that question by stating that a system gains or loses kinetic energy by transferring it through work between the environment (forces being introduced into the system) and the system. In basic terms, doing positive work means increasing kinetic energy.

Work Is Less Work
In some situations, using the Big Five Equations is easier. However, in several instances, skipping the Big Five and using work and energy can make problems much simpler. This is because we do not have to worry about the directional part of vectors and deal only with scalar quantities.

Example 6 What is the kinetic energy of a ball (mass = 0.10 kg) moving with a speed of 30 m/s?

Solution. From the definition,

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(0.10\text{ kg})(30\text{ m/s})^2 = 45\text{ J}$$

Example 7 A tennis ball (mass = 0.06 kg) is hit straight upward with an initial speed of 50 m/s. How high would it go if air resistance were negligible?

Solution. This could be done using the Big Five, but let's try to solve it using the concepts of work and energy. As the ball travels upward, gravity acts on it by doing negative work. [The work is negative because gravity is opposing the upward motion. F_w and d are in opposite directions, so θ = 180°, which tells us that $W = (F_w \cos \theta)d = -F_w d$.] At the moment the ball reaches its highest point, its speed is 0, so its kinetic energy is also 0. The Work–Energy Theorem says

$$W = \Delta K \quad \Rightarrow \quad -F_w d = 0 - \frac{1}{2}mv_0^2 \quad \Rightarrow \quad d = \frac{\frac{1}{2}mv_0^2}{F_w} = \frac{\frac{1}{2}mv_0^2}{mg} = \frac{\frac{1}{2}v_0^2}{g} = \frac{\frac{1}{2}(50 \text{ m/s})^2}{10 \text{ m/s}^2} = 125 \text{ m}$$

While the solutions in this book generally present just one method of obtaining the correct answer, that doesn't mean it's the only way. Don't worry about solving a problem the "wrong" way for the free-response questions. As long as your solution is valid, you'll get full credit, even if it isn't the simplest method.

Example 8 Consider the box sliding down the inclined plane in Example 4. If it starts from rest at the top of the ramp, with what speed does it reach the bottom?

Solution. It was calculated in Example 4 that W_{total} = 1,019 J. According to the Work–Energy Theorem,

$$W_{\text{total}} = \Delta K \quad \Rightarrow \quad W_{\text{total}} = K_f - K_i = K_f = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{\frac{2W_{\text{total}}}{m}} = \sqrt{\frac{2(1{,}019 \text{ J})}{35 \text{ kg}}} = 7.6 \text{ m/s}$$

Example 9 A pool cue striking a stationary billiard ball (mass = 0.25 kg) gives the ball a speed of 2 m/s. If the average force of the cue on the ball was 200 N, over what distance did this force act?

Solution. The kinetic energy of the ball as it leaves the cue is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(0.25 \text{ kg})(2 \text{ m/s})^2 = 0.50 \text{ J}$$

The work (W) done by the cue gave the ball this kinetic energy, so

$$W = \Delta K \quad \Rightarrow \quad W = K_f \quad \Rightarrow \quad Fd = K \quad \Rightarrow \quad d = \frac{K}{F} = \frac{0.50 \text{ J}}{200 \text{ N}} = 0.0025 \text{ m} = 0.25 \text{ cm}$$

POTENTIAL ENERGY

Kinetic energy is the energy an object has by virtue of its motion. Potential energy is independent of motion; it arises from the object's position (or the system's configuration). For example, a ball at the edge of a tabletop has energy that could be transformed into kinetic energy if it falls off. An arrow in an archer's pulled-back bow has energy that could be transformed into kinetic energy if the archer releases the arrow. Both of these examples illustrate the concept of **potential energy,**the energy an object or system has by virtue of its position or configuration. In each case, work was done on the object to put it in the given configuration (the ball was lifted to the tabletop, the bowstring was pulled back), and since work is the means of transferring energy, these things have stored energy that can be retrieved as kinetic energy. Also remember that potential energy can be found in multiple sources such as chemical sources, mechanical sources, and objects in gravitational fields. This is potential energy, denoted by U.

Because there are different types of forces, there are different types of potential energy. The ball at the edge of the tabletop provides an example of **gravitational potential energy,** U_g, which is the energy stored by virtue of an object's position in a gravitational field. This energy would be converted to kinetic energy as gravity pulled the ball down to the floor. For now, let's concentrate on gravitational potential energy.

Assume the ball has a mass m of 2 kg and that the tabletop is h = 1.5 m above the floor. How much work did gravity do as the ball was lifted from the floor to the table? The strength of the gravitational force on the ball is $F_w = mg = (2\text{ kg})(10\text{ N/kg}) = 20\text{ N}$. The force $\mathbf{F}_w$ points downward, and the ball's motion was upward, so the work done by gravity during the ball's ascent was

$$W_{\text{by gravity}} = -F_w h = -mgh = -(20\text{ N})(1.5\text{ m}) = -30\text{ J}$$

So someone performed +30 J of work to raise the ball from the floor to the tabletop. That energy is now stored, and if the ball were given a push to send it over the edge, by the time the ball reached the floor, it would acquire a kinetic energy of 30 J. We therefore say that the change in the ball's gravitational potential energy in moving from the floor to the table was +30 J. That is,

$$\boxed{\Delta U_g = -W_{\text{by gravity}}}$$

Who Cares About Potential Energy?
What if an object were to never fall? Does it even matter if we calculate its potential energy? Calculating potential energy at a point does not tell us much. We only care about potential energy only when we make it change from one point to another point. This in turn translates into doing work (increasing kinetic energy).

Note that potential energy, like work (and kinetic energy), is expressed in joules.

In general, if an object of mass m is raised a height h (which is small enough that g stays essentially constant over this altitude change), then the increase in the object's gravitational potential energy is

$$\boxed{\Delta U_g = mgh}$$

An important fact that makes the above equation possible is that the work done by gravity as the object is raised does not depend on the path taken by the object. The ball could be lifted straight upward or in some curvy path; it would make no difference. Gravity is said to be a **conservative**force because of this property. Conversely, any work done by a nonconservative force is path-dependent. Such is the case for friction and air resistance, as different paths taken may require more or less work to get from the initial to final positions.

One way to distinguish between conservative and nonconservative forces is that nonconservative forces do NOT exist in ideal situations.

If we decide on a reference level to call $h = 0$, then we can say that the gravitational potential energy of an object of mass m at a height h is $U_g = mgh$. In order to use this last equation, it's essential that we choose a reference level for height. For example, consider a passenger in an airplane reading a book. If the book is 1 m above the floor of the plane, then, to the passenger, the gravitational potential energy of the book is mgh, where $h = 1$ m. However, to someone on the ground looking up, the floor of the plane may be, say, 9,000 m above the ground. So, to this person, the gravitational potential energy of the book is mgH, where $H = 9{,}001$ m. What both would agree on, though, is that the difference in potential energy between the floor of the plane and the position of the book is $mg \times (1\text{ m})$, since the airplane passenger would calculate the difference as $mg \times (1\text{ m} - 0\text{ m})$, while the person on the ground would calculate it as $mg \times (9{,}001\text{ m} - 9{,}000\text{ m})$. Differences, or changes, in potential energy are unambiguous, but values of potential energy are relative.

Example 10 A stuntwoman (mass = 60 kg) scales a 40-meter-tall rock face. What is her gravitational potential energy (relative to the ground)?

Solution. Calling the ground $h = 0$, we find

$$U_g = mgh = (60\text{ kg})(10\text{ m/s}^2)(40\text{ m}) = 24{,}000\text{ J}$$

Example 11 If the stuntwoman in the previous example were to jump off the cliff, what would be her final speed as she landed on a large, air-filled cushion lying on the ground?

Solution. The gravitational potential energy would be transformed into kinetic energy. So

$$U \to K \quad \Rightarrow \quad U \to \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{\frac{2 \cdot U}{m}} = \sqrt{\frac{2(24{,}000\text{ J})}{60\text{ kg}}} = 28\text{ m/s}$$

CONSERVATION OF MECHANICAL ENERGY

We have seen energy in its two basic forms: kinetic energy (K) and potential energy (U). The sum of an object's kinetic and potential energies is called its **total mechanical energy,** E.

$$E = K + U$$

(Note that because U is relative, so is E.)

A Note About Kinetic and Potential Energy

Kinetic energy and potential energy have an inverse relationship. As kinetic energy increases, potential energy decreases and vice-versa.

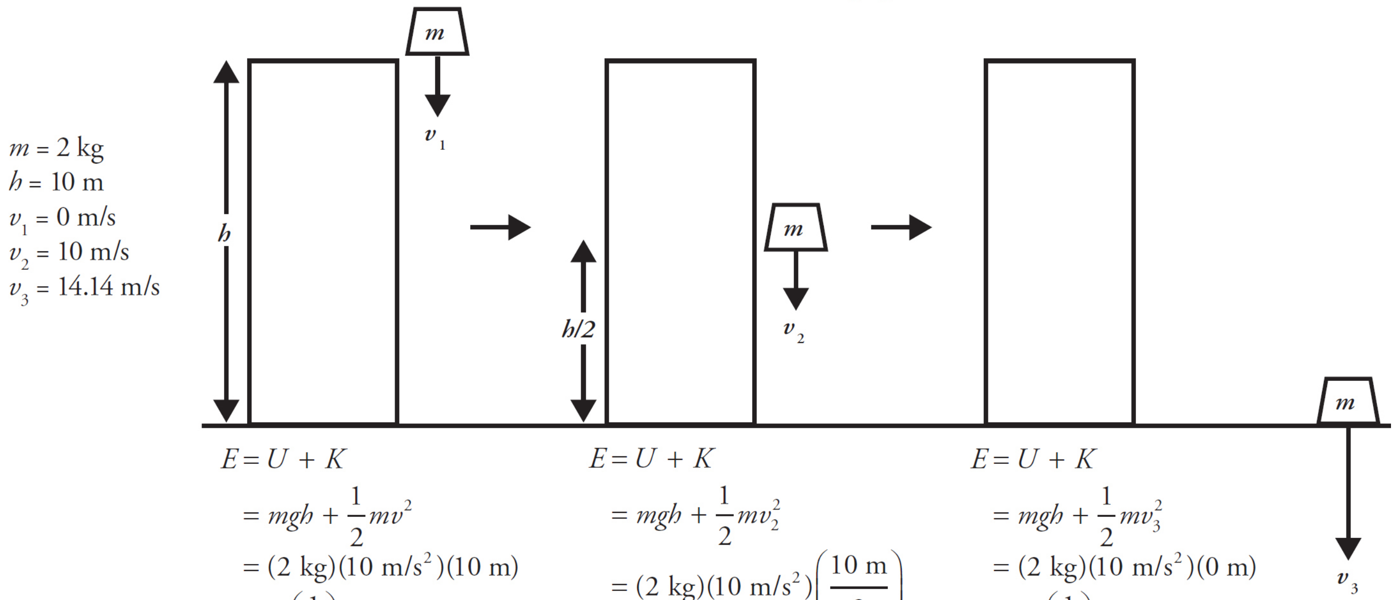
Assuming that no nonconservative forces (friction, for example) act on an object or system while it undergoes some change, then mechanical energy is conserved. Mechanical energy, that is the sum of potential and kinetic energies, is dissipated or converted into other energy forms such as heat, by nonconservative forces. So, if there are no nonconservative forces acting on the system, the initial mechanical energy, E_i, is equal to the final mechanical energy, E_f, or

$$K_i + U_i = K_f + U_f$$

This is the simplest form of the **Law of Conservation of Total Energy.**

Let's evaluate a situation in which an object, initially at rest, is falling from a building and calculate the total mechanical energy in each situation:

Note: Assume air resistance is negligible.



$$\begin{aligned} E &= U + K \\ &= mgh + \frac{1}{2}mv^2 \\ &= (2\text{ kg})(10\text{ m/s}^2)(10\text{ m}) \\ &\quad + \left(\frac{1}{2}\right)(2\text{ kg})(0\text{ m/s}) \\ &= 200\text{ J} + 0\text{ J} \\ &= 200\text{ J} \end{aligned}$$

$$\begin{aligned} E &= U + K \\ &= mgh + \frac{1}{2}mv_2^2 \\ &= (2\text{ kg})(10\text{ m/s}^2)\left(\frac{10\text{ m}}{2}\right) \\ &\quad + \left(\frac{1}{2}\right)(2\text{ kg})(10\text{ m/s})^2 \\ &= 100\text{ J} + 100\text{ J} \\ &= 200\text{ J} \end{aligned}$$

$$\begin{aligned} E &= U + K \\ &= mgh + \frac{1}{2}mv_3^2 \\ &= (2\text{ kg})(10\text{ m/s}^2)(0\text{ m}) \\ &\quad + \left(\frac{1}{2}\right)(2\text{ kg})(14.14\text{ m/s})^2 \\ &= 0\text{ J} + 200\text{ J} \\ &= 200\text{ J} \end{aligned}$$

What can you say about the calculated total mechanical energy for each situation? They all are equal. An independent system obeys the Law of Conservation of Energy. Since the total mechanical energy remains the same throughout its travel, sometimes using energy to solve problems is a lot simpler than using kinematics.

Example 12 A ball of mass 2 kg is gently pushed off the edge of a tabletop that is 5.0 m above the floor. Find the speed of the ball as it strikes the floor.

Solution. Ignoring the friction due to the air, we can apply Conservation of Mechanical Energy. Calling the floor our h = 0 reference level, we write

$$
\begin{aligned}
K_i + U_i &= K_f + U_f \\
0 + mgh &= \frac{1}{2}mv^2 + 0 \\
v &= \sqrt{2gh} \\
&= \sqrt{2(10\ \text{m/s}^2)(5.0\ \text{m})} \\
&= 10\ \text{m/s}
\end{aligned}
$$

Note that the ball's potential energy decreased, while its kinetic energy increased. This is the basic idea behind Conservation of Mechanical Energy: one form of energy decreases while the other increases.

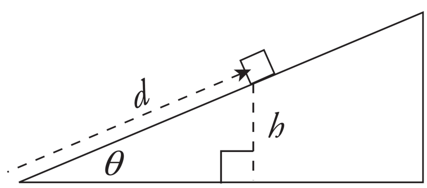
Example 13 A box is projected up a long ramp (incline angle with the horizontal = 37°) with an initial speed of 10 m/s. If the surface of the ramp is very smooth (essentially frictionless), how high up the ramp will the box go? What distance along the ramp will it slide?

Solution. Because friction is negligible, we can apply Conservation of Mechanical Energy. Calling the bottom of the ramp our h = 0 reference level, we write

$$
\begin{aligned}
K_i + U_i &= K_f + U_f \\
\frac{1}{2}mv_0^2 + 0 &= 0 + mgh \\
h &= \frac{\frac{1}{2}v_0^2}{g} \\
&= \frac{\frac{1}{2}(10\ \text{m/s})^2}{10\ \text{m/s}^2} \\
&= 5\ \text{m}
\end{aligned}
$$

Energy In = Energy Out
The Law of Conservation of Energy states that all the energy that was added to the universe during the big bang is conserved and energy cannot be added or removed from the universe. This means that all energy added to any system must come out in some form. For example, all of the energy used to bring a roller coaster to the top of the first hill is equal to all of the energy required to bring the roller coaster through all of the twists and loops and back to the starting point. Energy in this system is converted into other forms, but the total input of energy will always equal the total output.

Since the incline angle is θ = 37°, the distance d it slides up the ramp is found in this way:



$$
\begin{aligned}
h &= d\sin\theta \\
d &= \frac{h}{\sin\theta} = \frac{5\ \text{m}}{\sin 37^\circ} = \frac{25}{3}\ \text{m} = 8.3\ \text{m}
\end{aligned}
$$

Example 14 A skydiver jumps from a hovering helicopter that's 3,000 meters above the ground. If air resistance can be ignored, how fast will he be falling when his altitude is 2,000 m?

Solution. Ignoring air resistance, we can apply Conservation of Mechanical Energy. Calling the ground our h = 0 reference level, we write

$$\begin{aligned} K_\text{i} + U_\text{i} &= K_\text{f} + U_\text{f} \\ 0 + mgH &= \frac{1}{2}mv^2 + mgh \\ v &= \sqrt{2g(H-h)} \\ &= \sqrt{2(10 \text{ m/s}^2)(3{,}000 \text{ m} - 2{,}000 \text{ m})} \\ &= 140 \text{ m/s} \end{aligned}$$

CONSERVATION OF ENERGY WITH NONCONSERVATIVE FORCES

The equation $K_\text{i} + U_\text{i} = K_\text{f} + U_\text{f}$ holds if no nonconservative forces are doing work. However, if work is done by such forces during the process under investigation, then the equation needs to be modified to account for this work as follows:

$$K_\text{i} + U_\text{i} + W_\text{other} = K_\text{f} + U_\text{f}$$

Example 15 Wile E. Coyote (mass = 40 kg) falls off a 50-meter-high cliff. On the way down, the force of air resistance has an average strength of 100 N. Find the speed with which he crashes into the ground.

Why on the Left and Not the Right?
Work done by nonconservative forces is placed on the initial energy side because the final energy accounts for both the initial energy plus the energy that is dissipated by the object as it overcomes nonconservative forces.

Solution. The force of air resistance opposes the downward motion, so it does negative work on the coyote as he falls: $W_\text{r} = -F_\text{r}h$. Calling the ground h = 0, we find that

$$\begin{aligned} K_\text{i} + U_\text{i} + W_\text{r} &= K_\text{f} + U_\text{f} \\ 0 + mgh + (-F_\text{r}h) &= \frac{1}{2}mv^2 + 0 \\ v &= \sqrt{2h(g - F_\text{r}/m)} = \sqrt{2(50)(10 - 100/40)} = 27 \text{ m/s} \end{aligned}$$

Example 16 A skier starts from rest at the top of a 20° incline and skis in a straight line to the bottom of the slope, a distance d (measured along the slope) of 400 m. If the coefficient of kinetic friction between the skis and the snow is 0.2, calculate the skier's speed at the bottom of the run.

Solution. The strength of the friction force on the skier is $F_f = \mu_k F_N = \mu_k(mg \cos \theta)$, so the work done by friction is $-F_f d = \mu_k(mg \cos \theta)\cdot d$. The vertical height of the slope above the bottom of the run (which we designate the $h = 0$ level) is $h = d \sin \theta$. Therefore, Conservation of Mechanical Energy (including the negative work done by friction) gives

$$\begin{aligned}
K_i + U_i + W_{friction} &= K_{ff} + U \\
0 + mgh + (-\mu_k mg \cos \theta \cdot d) &= \tfrac{1}{2} mv^2 + 0 \\
mg(d \sin \theta) + (-\mu_k mg \cos \theta \cdot d) &= \tfrac{1}{2} mv^2 \\
gd(\sin \theta - \mu_k \cos \theta) &= \tfrac{1}{2} v^2 \\
v &= \sqrt{2gd(\sin \theta - \mu_k \cos \theta)} \\
&= \sqrt{2(10)(400)[\sin 20° - (0.2) \cos 20°]} \\
&= 35 \text{ m/s}
\end{aligned}$$

Working Backward to Go Forward

Working with energy does not take vectors into account. With many problems, we can solve them backwards with Conservation of Energy in order to find out the initial energy, work, or final energy; and in turn, we can solve for other useful information such as height or speed.

So far, any of the problems we have solved in this chapter could have been solved using the kinematics equations and Newton's laws. The truly powerful thing about energy is that in a closed system, changes in energy are independent of the path you take. This allows you to solve many problems, you would not otherwise be able to solve. With many energy problems, you do not need to measure time with a stopwatch, you do not need to know the mass of the object, you do not need a constant acceleration (remember that is required for our Big Five equations from kinematics), and you do not need to know the path the object takes.

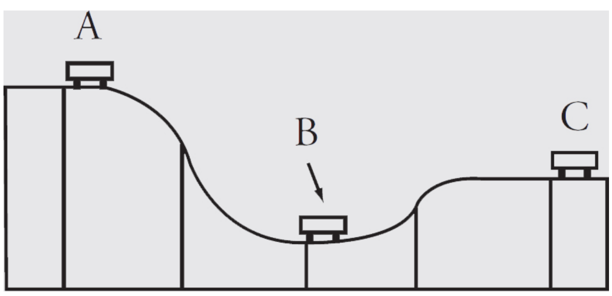
Time Is of No Difference (And Many Others)

Energy is almost always the easier approach than kinematics. We can throw out so many variables with energy.

Example 17 A roller coaster at an amusement park is at rest on top of a 30 m hill (point A). The car starts to roll down the hill and reaches point B, which is 10 m above the ground, and then rolls up the track to point C, which is 20 m above the ground.

(a) A student assumes no energy is lost, and solves for how fast is the car moving at point C using energy arguments. What answer does he get?

(b) If the final speed at C is actually measured to be 2 m/s, where did the lost energy go?



Solution.

(a) Our standard energy equation states

$$K_i + U_i = K_f + U_f$$

or

$$\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f$$

Canceling the mass, setting $v_i = 0$ m/s, and rearranging terms, we get

$$v_f = \sqrt{2g\Delta h}$$

$$v_f = \sqrt{21(10 \text{ m/s}^2)(30 \text{ m} - 20 \text{ m})}$$

$$v_f = \sqrt{200 \text{ m}^2/\text{s}^2} = 10\sqrt{2} \text{ m/s}$$

(b) The lost energy was likely lost as heat.

POWER

Simply put, **power** is the rate at which work gets done (or energy gets transferred, which is the same thing). Suppose Scott and Jean each do 1,000 J of work, but Scott does the work in 2 minutes, while Jean does it in 1 minute. They both did the same amount of work, but Jean did it more quickly; thus Jean was more powerful. Here's the definition of power:

$$\text{Power} = \frac{\text{Work}}{\text{time}} \quad \text{—in symbols} \rightarrow \quad P = \frac{W}{t}$$

The unit of power is the joule per second (J/s), which is renamed the **watt,** and symbolized W (not to be confused with the symbol for work, *W*). One watt is 1 joule per second: 1 W = 1 J/s. Here in the United States, which still uses older units like inches, feet, yards, miles, ounces, pounds, and so forth, you still hear of power ratings (particularly of engines) expressed in horsepower. One horsepower is defined as 1 hp = 746 W.

Note that this conversion will be provided on the test.

$P = W/t$	Recall: $W = Fd$
$P = Fd/t$	Recall: $v = d/t$

$$P = W/t = Fd/t = Fv$$

This equation only applies for a constant force parallel to a constant velocity. Remember to check that your equation fits the given circumstances!

Example 18 A mover pushes a large crate (mass m = 75 kg) from the inside of the truck to the back end (a distance of 6 m), exerting a steady push of 300 N. If he moves the crate this distance in 20 s, what is his power output during this time?

Solution. The work done on the crate by the mover is $W = Fd$ = (300 N)(6 m) = 1,800 J. If this much work is done in 20 s, then the power delivered is $P = W/t$ = (1,800 J)/(20 s) = 90 W.

Example 19 What must be the power output of an elevator motor that can lift a total mass of 1,000 kg and give the elevator a constant speed of 8.0 m/s?

Solution. The equation $P = Fv$, with $F = mg$, yields

$$P = mgv = (1{,}000 \text{ kg})(10 \text{ N/kg})(8.0 \text{ m/s}) = 80{,}000 \text{ W} = 80 \text{ kW}$$

Study Break

Phew! You have been working hard through these content chapters! Wrap up Chapter 6 and then give yourself a break—go for a walk, take a dance break, get a snack—before you continue on.

Summary

- Work is force applied across a displacement. Work can cause a change in energy. Positive work puts energy into a system, while negative work takes energy out of a system. Basic equations for work include

$$W = Fd\cos\theta$$

$$\text{Work} = \Delta KE$$

$$W = \text{area under an } F\text{-versus-}d \text{ graph}$$

- Energy is a conserved quantity. By that we mean the total initial energy is equal to the total final energy. Basic equations with energy include

$$K = \frac{1}{2}mv^2$$

$$U_g = mgh$$

- Often, we limit ourselves to mechanical energy with no heat lost or gained. In this case,

$$K_i + U_i \pm W = K_f + U_f$$

- Power is the rate at which one does work and is given by

$$P = \frac{W}{t} \text{ or } P = Fv$$