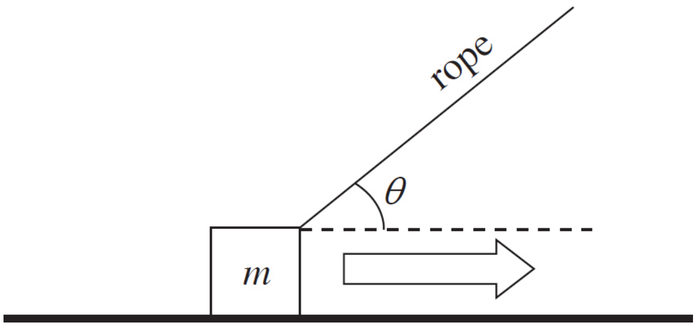
Section II: Free Response

1. This question concerns the motion of a crate being pulled across a horizontal floor by a rope. In the diagram below, the mass of the crate is m, the coefficient of kinetic friction between the crate and the floor is μ, and the tension in the rope is $\mathbf{F_T}$.

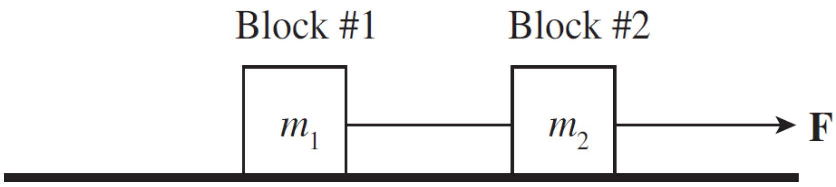


(a) Draw and label all of the forces acting on the crate.

(b) Compute the normal force acting on the crate in terms of m, F_T, θ, and g.

(c) Compute the acceleration of the crate in terms of m, F_T, θ, μ, and g.

2. In the diagram below, a massless string connects two blocks—of masses m_1 and m_2, respectively—on a flat, frictionless tabletop. A force $\mathbf{F}$ pulls on Block #2, as shown:



(a) Draw and label all of the forces acting on Block #1.

(b) Draw and label all of the forces acting on Block #2.

(c) What is the acceleration of Block #1? Please state your answer in terms of F, m_1, and m_2.

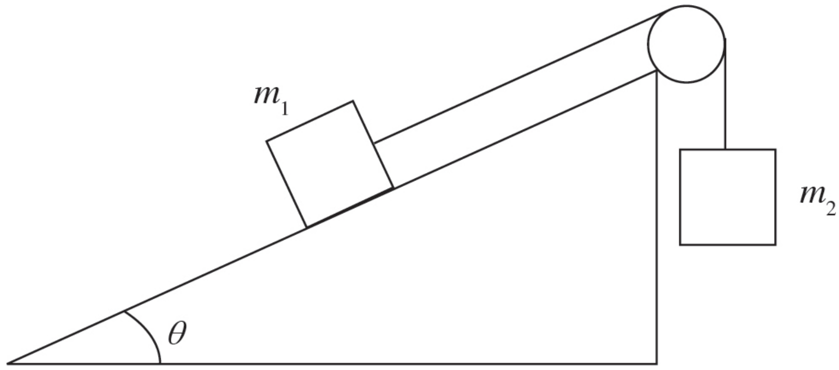
(d) What is the tension in the string connecting the two blocks? Please state your answer in terms of F, m_1, and m_2.

(e) If the string connecting the blocks were not massless, but instead had a mass of m, determine

(i) the acceleration of Block #1, in terms of F, m, m_1, and m_2.

(ii) the difference between the strength of the force that the connecting string exerts on Block #2 and the strength of the force that the connecting string exerts on Block #1. Please state your answer in terms of F, m, m_1, and m_2.

3. In the figure shown, assume that the pulley is frictionless and massless.



(a) If the surface of the inclined plane is frictionless, determine what value(s) of θ will cause the box of mass m_1 to

(i) accelerate up the ramp

(ii) slide up the ramp at constant speed

(b) If the coefficient of kinetic friction between the surface of the inclined plane and the box of mass m_1 is μ_k, derive (but do not solve) an equation satisfied by the value of θ, which will cause the box of mass m_1 to slide up the ramp at constant speed.

4. A skydiver is falling with speed v_0 through the air. At that moment (time $t = 0$), she opens her parachute and experiences the force of air resistance whose strength is given by the equation $F = kv$, in which k is a proportionality constant and v is her descent speed. The total mass of the skydiver and equipment is m. Assume that g is constant throughout her descent.

(a) Draw and label all the forces acting on the skydiver after her parachute opens.

(b) Determine the skydiver's acceleration in terms of m, v, k, and g.

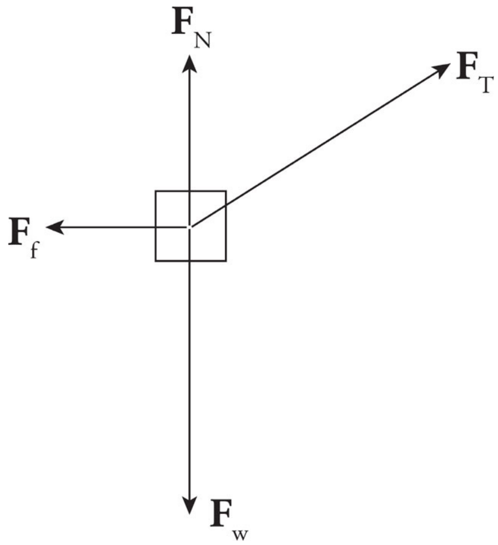
(c) Determine the skydiver's terminal speed (that is, the eventual constant speed of descent).

(d) Sketch a graph of v as a function of time, being sure to label important values on the vertical axis.

Section II: Free Response

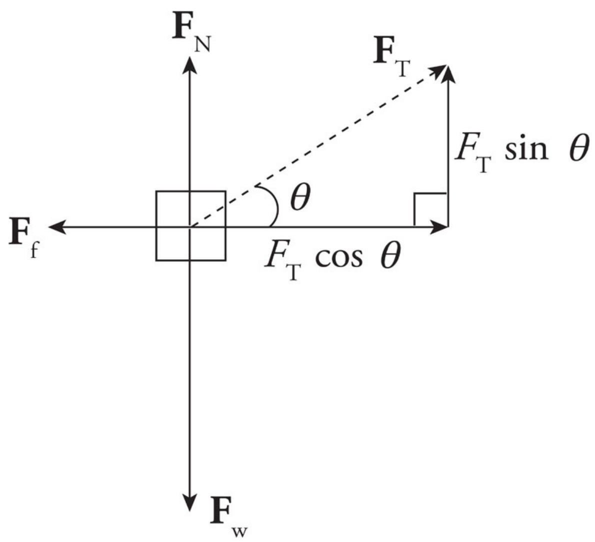
1. (a)

The forces acting on the crate are $\mathbf{F}_T$ (the tension in the rope), $\mathbf{F}_w$ (the weight of the block), $\mathbf{F}_N$ (the normal force exerted by the floor), and $\mathbf{F}_f$ (the force of kinetic friction):



(b)

First, break $\mathbf{F}_T$ into its horizontal and vertical components:



Since the net vertical force on the crate is zero, you get $F_N + F_T \sin\theta = F_w$, so $F_N = F_w - F_T \sin\theta = mg - F_T \sin\theta$.

(c)

From part (b), notice that the net horizontal force acting on the crate is

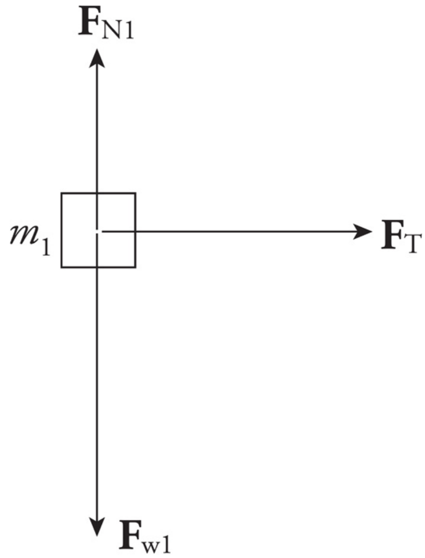
$$F_T \cos\theta - F_f = F_T \cos\theta - \mu F_N = F_T \cos\theta - \mu(mg - F_T \sin\theta)$$

so the crate's horizontal acceleration across the floor is

$$a = \frac{F_{net}}{m} = \frac{F_T \cos\theta - \mu(mg - F_T \sin\theta)}{m}$$

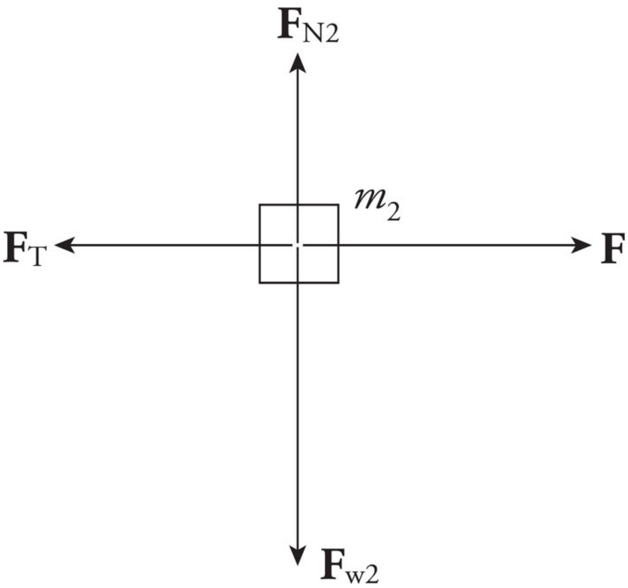
2. (a)

The forces acting on Block #1 are $\mathbf{F}_T$ (the tension in the string connecting it to Block #2), $\mathbf{F}_{w1}$ (the weight of the block), and $\mathbf{F}_{N1}$ (the normal force exerted by the tabletop):



(b)

The forces acting on Block #2 are **F** (the pulling force), $\mathbf{F}_T$ (the tension in the string connecting it to Block #1), $\mathbf{F}_{w2}$ (the weight of the block), and $\mathbf{F}_{N2}$ (the normal force exerted by the tabletop):



(c)

Newton's Second Law applied to Block #2 yields $F - F_T = m_2a$, and applied to Block #1 yields $F_T = m_1a$. Adding these equations, you get $F = (m_1 + m_2)a$, so

$$a = \frac{F}{m_1 + m_2}$$

(d)

Substituting the result of part (c) into the equation $F_T = m_1a$, you get

$$F_T = m_1 a = \frac{m_1}{m_1 + m_2} F$$

(e)

(i) Since the force **F** must accelerate all three masses—m_1, m, and m_2—the common acceleration of all parts of the system is

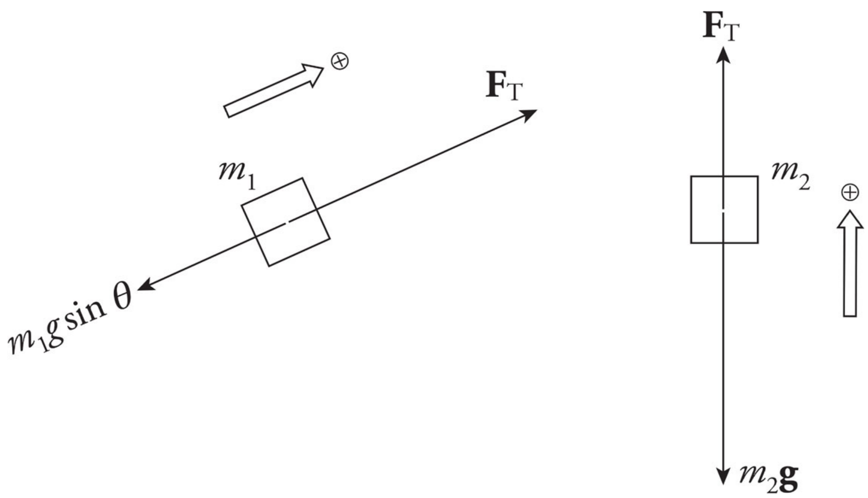
$$a = \frac{F}{m_1 + m + m_2}$$

(ii) Let $\mathbf{F}_{T1}$ denote the tension force in the connecting string acting on Block #1, and let $\mathbf{F}_{T2}$ denote the tension force in the connecting string acting on Block #2. Then, Newton's Second Law applied to Block #1 yields $F_{T1} = m_1 a$ and applied to Block #2 yields $F - F_{T2} = m_2 a$. Therefore, using the value for a computed above, you get

$$\begin{aligned} F_{T2} - F_{T1} &= (F - m_2 a) - m_1 a \\ &= F - (m_1 + m_2)a \\ &= F - (m_1 + m_2)\frac{F}{m_1 + m + m_2} \\ &= F\left(1 - \frac{m_1 + m_2}{m_1 + m + m_2}\right) \\ &= F\frac{m}{m_1 + m + m_2} \end{aligned}$$

3. (a)

First, draw free-body diagrams for the two boxes:



Applying Newton's Second Law to the boxes yields the following two equations:

$F_T - m_1 g \sin\theta = m_1 a$ (1)

$F_T - m_2 g = m_2(-a)$ (2)

Subtract the equations and solve for a:

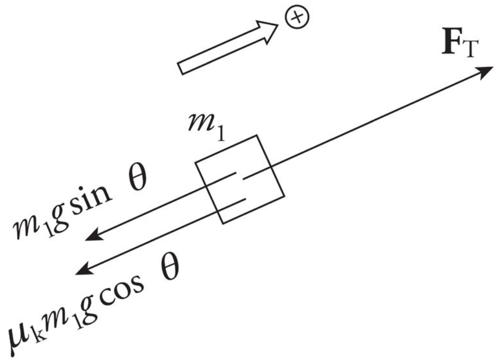
$$\begin{aligned} m_2 g - m_1 g \sin\theta &= (m_1 + m_2)a \\ a &= \frac{m_2 - m_1 \sin\theta}{m_1 + m_2} g \end{aligned}$$

(i) For a to be positive, you must have $m_2 - m_1 \sin\theta > 0$, which implies that $\sin\theta < m_2/m_1$, or, equivalently, $\theta < \sin^{-1}(m_2/m_1)$.

(ii) For a to be zero, you must have $m_2 - m_1 \sin\theta = 0$, which implies that $\sin\theta = m_2/m_1$, or, equivalently, $\theta = \sin^{-1}(m_2/m_1)$.

(b)

Including the force of kinetic friction, the force diagram for m_1 is



Since $F_f = \mu_k F_N = \mu_k m_1 g \cos\theta$, applying Newton's Second Law to the boxes yields these two equations:

$F_T - m_1 g \sin\theta - \mu_k mg \cos\theta = m_1 a$ (1)

$m_2 g - F_T = m_2 a$ (2)

Add the equations and solve for a:

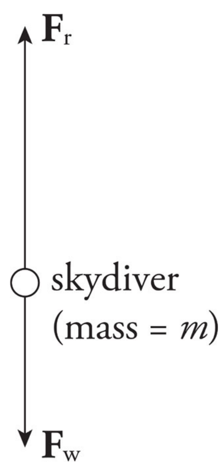
$$m_2 g - m_1 g \sin\theta - \mu_k mg\cos\theta = (m_1 + m_2)a$$
$$a = \left(\frac{m_2 - m_1(\sin\theta + \mu_k\cos\theta)}{m_1 + m_2}\right)g$$

In order for a to be equal to zero (so that the box of mass m_1 slides up the ramp with constant velocity),

$$m_2 - m_1(\sin\theta + \mu_k \cos\theta) = 0$$
$$\sin\theta + \mu_k\cos\theta = \frac{m_2}{m_1}$$

4. (a)

The forces acting on the skydiver are $\mathbf{F}_r$, the force of air resistance (upward), and $\mathbf{F}_w$, the weight of the skydiver (downward):



(b)

Since $F_{net} = F_w - F_r = mg - kv$, the skydiver's acceleration is

$$a = \frac{F_{net}}{m} = \frac{mg - kv}{m}$$

(c)

Terminal speed occurs when the skydiver's acceleration becomes zero, since then the descent velocity becomes constant. Setting the expression derived in part (b) equal to 0, find the speed $v = v_t$ at which this occurs:

$$v = v_t \text{ when } a = 0 \quad \Rightarrow \quad \frac{mg - kv_t}{m} = 0 \quad \Rightarrow \quad v_t = \frac{mg}{k}$$

(d)

The skydiver's descent speed is initially v_0 and the acceleration is (close to) g. However, once the parachute opens, the force of air resistance provides a large (speed-dependent) upward acceleration, causing her descent velocity to decrease. The slope of the velocity-versus-time graph (the acceleration) is not constant but instead decreases to zero as her descent speed decreases from v_0 to v_t. Therefore, the graph is not linear.